Aristotle on non-contradiction

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1. A modern work on the principle of non-contradiction (from now PNC) would usually be considered part of the study of logic. This last discipline is defined by Immanuel Kant as an “a priori science of the necessary laws of thought, but not considering particular objects, though all objects in general, from intellect to reason, but not in a subjective meaning, as to say by the empirical (psychological) principles, following which the intellect thinks, but in an objective meaning, as to say by the a priori principles according to which it has to think”\(^1\). Logic would be, therefor, a science, whose object is the system of the necessary laws of thought, independently from the object of thought itself\(^2\). Aristotle did not use the word “logic” in the same way as we do\(^3\), but he was, nevertheless, the creator of what we call formal logic, a discipline that he called *analytics*\(^4\).

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\(^2\) As in modern logic handbooks: see also M. Malatesta, *La logica primaria*, LER, Roma 1988, p. 51: “Noi moderni definiamo le leggi logiche o tautologie come enunciati molecolari [...] sempre veri, veri in tutti i mondi possibili, cioè veri indipendentemente tanto dal senso dei singoli enunciati, quanto dal valore di verità di essi. Se poi le leggi logiche o tautologie siano leggi immutabili ed eterne che l'intelletto umano scopre ma non crea, o leggi della mente, o l'una e l'altra cosa, o in parte l'una cosa, in parte l'altra, o altro ancora, e una questione di filosofia della logica che non intacca l'oggetto della logica sul quale tutti i logici contemporanei convengono”; E. Agazzi, *La logica simbolica*, La Scuola, 1991: “...la logica si disinteressa quasi completamente del 'contenuto' dei discorsi argomentativi”.


\(^4\) See also A. Jori, *cit.* p. 54; *Rhet* 1359b27-28. But observing that the name “Analytics” given to the works about syllogisms is sufficient.
And though Aristotelian analytics are essentially a theory of syllogism and a study of the links between propositions: what is the place of the PNC in this context? Answering this question will make clear the meaning, and the value, of that which the Stagirite called the “firmest principle”; if this is only a law of the logos, or if it’s part of a greater level of his doctrine. To this aim, I’ll take into consideration the context in which the PNC is treated by Aristotle, and the meaning, to him, of the link between logic and ontology, thought and reality.

1.1 The PNC frequently appears in Aristotle’s “logic” works, collected under the name of *Organon*. But why does the Stagirite use it, how does he use it?

In *De Interpretatione* it can be found, expressively affirmed, two times: once in the twelfth chapter, dedicated to the study of opposition between modal propositions: another one in the fourteenth chapter, that develops the relation of contrariety between opinions.

In the first case Aristotle wants to establish which is the contradictory proposition of the phrase “it’s possible that it is”. This is his procedure: a) what is possible to be, is also possible not to be b) “it’s possible not to be” seems to be the negation of “it’s possible to be” c) but, if these two propositions were contradictory, then two contradictory propositions would be at the same time true, and that’s impossible for the PNC d) so, the contradictory of the analysed proposition is “it’s not possible, that it is”. Here the PNC is used to exclude the chance, for a proposition, to be considered contradictory of another one.

In the second case Aristotle uses the PNC to show a) how two contrary opinions cannot be true together: in fact contrary opinions are about the same subject, and it’s impossible for both the contraries to be predicated of the same b) how it’s not possible for the same man to have simultaneously contrary opinions. This passage

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5 *De int.* 21b17-18: “ἄλλα μὴν ἀδύνατον κατά τοῦ ἀντιοδιδόται τὰς ἀντικατάστασις φάσεις”

6 *De int.* 24b9: “ἀμα δὲ οὐκ ἐνδέχεται τὰ ἑναντία ὑπάρχειν τῷ αὐτῷ”; It’s the statement, with which the text ends.
will be considered again\textsuperscript{7}, but for now I want to remark how in this text is attested already an ontological and not just logical use of PNC.

In the \textit{Categories}, again, Aristotle uses the PNC in various occasions: he demonstrates\textsuperscript{8} the impossibility to consider Great and Little as contraries, because that would mean that the same subject can admit simultaneously both contraries; he states it again treating the different kinds of opposition when he considers affirmation and negation\textsuperscript{9}.

In the \textit{First Analytics} the PNC is expressly used to show, in various places of the text, the necessity of some implications between propositions\textsuperscript{10}: establishing the dependence of B from A, if A is possible also B would. “If in fact it was impossible, the same object would result simultaneously possible and impossible”\textsuperscript{11}, as to say possible because dependent from A, and impossible as established by the hypothesis. In the same way, later, in the second book, Aristotle states that it’s impossible to obtain false conclusions from true premises. If the two premises “A is predicated of every B” and “B is predicated of every C” are true, it’s necessary for the conclusion “A is predicated of every C” to be true as well, or “the same determination would belong and not belong to the same subject”\textsuperscript{12}. Here it’s possible to remark the strong relationship that (contrary to what do Lukasiewicz and I. Husic think\textsuperscript{13}) involves the syllogistic and the PNC, specifically the scientific syllogism and the PNC. The first one wants to express the necessary: but if the necessary is that whose negation is impossible, the impossible is the contradictory.

\textsuperscript{7} See also paragraph 4.4.
\textsuperscript{8} \textit{Cat.} 5b39-6a1: “ἀλλ’ οὐδὲν δοκεῖ ἅμα τὰ ἐναντία ἐπιδέχεσθαι”
\textsuperscript{9} \textit{Cat.} 13b2-3: “ἐπὶ μόνων γὰρ τούτων ἀναγκαῖον ἄει τὸ μὲν ἄληθες τὸ δὲ ψεῦδος αὐτῶν εἶναι”
\textsuperscript{10} \textit{An. Pr.} 34a25-30; 53b10-25
\textsuperscript{11} Aristotle, \textit{Organon}, edited by G. Colli, Adelphi, Milano 2003, p. 129
\textsuperscript{12} \textit{An. Pr.}, p. 206
\textsuperscript{13} See also J. Lukasiewicz, \textit{Il principio di contraddizione in Aristotele}, Quodlibet, Macerata, 2003, p.90
In other places of the same work Aristotle evokes the principle to solve some other little questions: at the end of the first book\footnote{An. Pr., pp. 196-202} some pages are written to clarify the relation of opposition to propositions. As already done in \textit{De interpretatione}, Aristotle shows the difference between “not being x” and “being non-x”: with the help of an example based on a modal proposition, he demonstrates that the negation of “being x” must be the first proposition and not the second one, or it could happen that contradictory propositions both be true.

He also uses the so-called “psychological formulation” of the PNC (which I’ll handle address later) to demonstrate the necessity of the relation of implication between opinions, as it happens with the corresponding statements.

At last, let’s remark on the strong relationship that links the PNC to the \textit{reductio ad absurdum} of the syllogism, where the premise is the contradictory to the conclusion of the syllogism that has to be demonstrated. A new premise is added, too. This will bring to the negation one of the premises of the first syllogism, and as this last premise is evident\footnote{An. Pr. 61b14} the negation of the conclusion of the first syllogism will per force be untrue. What sustains the demonstration is, as in the last cases, the impossibility for contradictory propositions to be considered both true\footnote{An. Pr. 61a18-35; see also An. Post. 77a23, where this relation is expressly affirmed.}.

In the \textit{Second analytics}, a treatise dedicated to the “logic of science”\footnote{A. Jori, \textit{Aristotele}, cit. p. 96}, Aristotle uses the PNC as an example because he’s trying to establish the nature of demonstration and to analyze the premises of scientific knowledge. In various places the PNC is used as an example of an “indemonstrable axiom”\footnote{An. Post. 71a14}; elsewhere it’s indicated by Aristotle as the axiom that no one science, in its own demonstration, can expressly assume.

In the Topics, the treatise dedicated to dialectics, Aristotle states that the chance to discuss, about a subject, the inherence or not of
a certain determination, implies the chance to discuss the inher-
ence of the contradictory determination too: for example, demon-
strating that a man is healthy means demonstrating that that man is
not sick, and in the same way demonstrating that a man is sick
means demonstrating that that man is not healthy.\footnote{19}

Let us remark how the PNC “resounds” in many places of the
book, in example at 137a7-20, where the distinctions and the im-
pirations that the dialectic process implies often recall the first
principle, even if not always in an explicit way. In the work’s last
pages\footnote{20} Aristotle numbers five ways in which a dialectic opponent
can ask to accept contradictory propositions, in order to keep the
reader warned about similar rhetorical tricks.

What can be stated, after this fast glance through the Organon? We
have seen how differently Aristotle uses the PNC, depending to
the argument and the aims he has at the moment. He uses the prin-
ciple to exclude absurd hypotheses, and to decide between various
alternatives; he uses it as an example when he talks about axioms
and indemonstrable principles; he uses it in the exposition of his
syllogistic and theory of demonstration; many times he uses it also
to analyze the opposition between propositions.

So it’s obvious, considering the “logical” context so far shown,
that the PNC appears strongly linked to formal problems: a valid
aid in the investigation of the laws of speech and argumentation.

But is that really so?

In Aristotle’s logical Essays it is already possible to contradict this
first impression: we’ve seen, in fact, that many times he applies
the PNC not only to propositions, but also to things and opinions
(fulfilling so the tripartition that J. Lukasiewicz has done of the
principle\footnote{21}).

We’ve seen, in fact, how in the Categories\footnote{22} Aristotle negates the
contrariety of Great and Small: it’s impossible, for the contraries,
to be applied at the same object, and because objects, that are at

\footnotetext[19]{Top. 112a25-13}
\footnotetext[20]{Top. 163a13-25}
\footnotetext[21]{See also chapter three.}
\footnotetext[22]{See also note 8.}
the same time small and great, exist, these two predicates cannot be contraries.

In *De interpretatione*\(^{23}\) is stated the impossibility of subsistence of contradictory opinions: also here the PNC hasn’t any logical value, but a psychological/ontological one. The same happens in the *First Analytics*\(^{24}\), and in a form that shows the not so strong distinction that Aristotle keeps between the various uses of the principle, and also his comfort in keeping together questions that we would today consider “logical”, and considerations that would be metaphysical or ontological.

Ultimately, it’s already possible to see how, to the Philosopher, the PNC is not only a law of thought and speech\(^{25}\). This is evident if we get to the most relevant moment of this question: the analysis of book IV of *Metaphysics*.

This book is famous because in it Aristotle defines first philosophy as ontology. This theme belongs, anyway, just to the first two, short chapters of the treatise, that continues with the explicit enunciation of the PNC and with its defense against eventual opponents. It cannot be said, however, that this book is divided in two different parts, nor that in it there are only the answers to particular questions of the third book\(^{26}\): what clearly appears is not the juxtaposition, but the strict interrelation between the different thematics of the work. That shows ultimately the nexus that links the Principle to First Philosophy, and subtracts it from any logical or dialectical interpretation.

The treatise begins recalling the science that contemplates “being as being”, and the properties that belong to it as itself\(^{27}\). This science is different from all the others, because these, after they have

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\(^{23}\) See also note 7.
\(^{24}\) 66b35-67a1-10.
\(^{25}\) It will soon be seen, how this distinction itself is, in a system how the one of Aristotle, acceptable only with some specifications.
\(^{27}\) *Met.* 1003a20-21 21: ”Ἔστιν ἐπιστήμη τις ἢ θεωρεῖ τὸ ὄν ἢ ὄν καὶ τὰ τοῦτο ὑπάρχοντα καθ’ αὐτόν”
taken a part of being, consider about it just what is accidental\(^{28}\) and not what belongs to it as being\(^{29}\). The ὁν ἔν ὁν is so the same as the ὁν κατά αὐτό: in both cases we refer to a reality considered “by itself”, in a universal mode\(^{30}\). The principles (ἀρχαί) and the causes (αἰτίαι) sought by this science concern all the entities, just because they “are”\(^{31}\).

Being is said in many senses\(^{32}\). The principal meaning of “being” is οὐσία\(^{33}\): all other things are called “entities” only “analogically”, because they’re related to it\(^{34}\).

So the relation between ontology and ousiology is traced: the philosopher looks for the causes and the principles of beings-qua-beings, and so it's his duty to investigate what mostly is, that which is οὐσία\(^{35}\).

The philosopher, so, shall study not only being, but also everything concerning the being as being: the one, the plural, negation and privation\(^{36}\), the identical and the different, contrariety\(^{37}\), anterior and posterior, the genre and the species\(^{38}\). All these properties are ἰδια of being as being: the PNC, which Aristotle finally

\(\text{Met. 1003a25: "περὶ τούτου θεωροῦσι τὸ συμβεβηκός."}\)

\(\text{Met. 1003a25-32}\)

\(\text{Met. 1003a32: "τὸ ὁν λέγεται πολλαχῶς."}\)

\(\text{Met. 1003b16-19: ousia is what “is first, that is what all the rest depends and is determined from”.}\)

\(\text{Met. 1003b5-11.}\)

\(\text{Met. 1003B19-21: further (1004a3-4) Aristotle also divides philosophy in as many parts as the substances. It’s evident that the precedent discourse has to be repeated, and this will happen in the passage 1005a33-b2, where Aristotle will specify how philosophy that studies the supreme genre of substance must study axioms too.}\)

\(\text{Met.1004a9-22}\)

\(\text{Met. 1004a27-28}\)

\(\text{Met. 1005a13-15}\)
introduces in the third chapter, is ἴδιον, or “proper”, of it too. Philosophy, in fact, must also study the so-called “axiomata”, because they concern all beings\textsuperscript{39}. On the other side, studying PNC is precisely first philosophy's duty. As we precedently had put οὐσία before other entities, now the supreme form of οὐσία is preferred to its inferior forms\textsuperscript{40}. But axiomata are “the surest principles of all beings-qua-beings” (περὶ τῶν ὄντων ἡ ὄντα τὰς πάντων βεβαιοτάτας, Met. 1005B10), and so their study will belong to the philosopher, who studies the eminent genre of substance. But here a mark is necessary: first of all, that οὐσία is “eminently” doesn't mean that she only is, and nothing more. Are “beings”, in fact, also all things as affection, corruptions, privations, qualities, causes that produce or generate οὐσία, objects who refer to οὐσία, negations of οὐσία or any other of these characteristics\textsuperscript{41}. PNC's field is not only οὐσία: it invests even not-being as negation of being\textsuperscript{42}.

Moreover, to privilege the supreme οὐσία rather than the other οὐσίαι doesn't mean that the PNC concerns her and not inferior οὐσίαι: as before, it's a necessary measure, taken to establish who has to study the principle itself, but it concerns separate οὐσίαι as the sensible ones, οὐσίαι as the properties of the οὐσία or their characteristics. But most of all, it has been seen how the principle is a feature of beings: how it can be counted among the objects of study of the philosopher, and specifically of the philosopher who studies being in its most eminent meaning. It's a principle of discourse and of thought, of reality and of argumentation\textsuperscript{43}: we just

\textsuperscript{39} Met. 10005a20-22;1005a27: “ὡσπερ ἔστι δὴν ὁ δὴν ὡσπερ ὅποιον ἔστιν ὧν ὕπορευε τὰ πᾶσα (τοῦτο γὰρ οὔτως τὸ κοινών), τοῖσι δὲ τῷ ὕπορευεν καὶ περὶ τούτων ἔστιν ἡ θεωρία”.

\textsuperscript{40} Met. 1005a34-38: “(forKey δὲ ἔστιν ἐν τῷ φυσικῷ τις ἀνωτέρω (ὅν γὰρ τι γένος τοῦ ὄντος ἡ φύσις), τοῖσι δὲ τοῦ καθόλου καὶ τοῖσι δὲ τῆς πρώτης οὐσίας καὶ ἡ περὶ τούτων ἐπὶ δεικτικός”.

\textsuperscript{41} Met. 1003b6-10.

\textsuperscript{42} see also E. Berti, Dalla dialettica alla filosofia prima, Bompiani, Milano 2004, p. 458 e G. Pasquale, Il principio di non-contraddizione in Aristotele, Bollati Boringhieri, Torino 2008, p. 27

\textsuperscript{43} Particularly interesting, on this topic, is the expression by Alexander from Aphrodisia: “what has to be possessed as principle of every knowable thing by
have to establish if to these different fields belong different principles, or the same.

1.2. Before we proceed to a more precise analysis of PNC and its formulations, it can be useful to premise a general remark on the nature of the relationship between logics and first philosophy in Aristotle's works, a remark already partially suggested in the last paragraph.

It has been seen, in fact, how Aristotle uses the Principle in the texts of the Organon, and how these can be considered works regarding pure logic. We've also seen how in this works it's already difficult to distinguish between a “logical” and an “ontological” use of PNC. In truth, what is problematic is the separation between logic and ontology itself, that belongs mostly to modern thought

Aristotle's philosophy belongs to the age of thought in which the identity between “sure” and “true” ruled: Greeks' realism consisted exactly in the lack of this fracture between thought and real, the order of the things and the order of the intellect (which, from an expression of Isaac Ben Salomon Israeli, will be called in the Middle Ages adaequatio rei et intellectus). Aristotle is placed in this spiritual atmosphere.

For this reason, for a pre-modern thinker as he is, the definition of logic that we've given, as a formal science of the laws of thought that abstracts radically from reality, cannot be valid: Aristotle's who wants to know any being” (Alessandro d'Afrodisia, Commentario alla Metafisica di Aristotele, a cura di G. Movia, Bompiani, Milano 2003, p. 635)

44 On this topic see also the famous work by Bonitz on the categories: H. Bonitz, Sulle categorie di Aristotele, Vita e pensiero, Milano 1995.

45 see also E. Severino, Filosofia, La scuola, Brescia 1991, pp. 6-7; but also G. W. F. Hegel, Scienza della logica, translated by A. Moni, Laterza, Roma-Bari 2008, pp. 25-26.

46 The expression that best expresses this can be found in De Int. 19a33: 19a33:“ὁμοίως οἱ λόγοι ἀληθεῖς ὡσπερ τὰ πράγματα”.

47 Düring, cit., p. 66: “In linea di massima, la parola “logico” significa in Aristotele che qualcosa viene discusso dal punto di vista linguistico-formale, e senza che si tenga conto del contenuto reale ”. It seems, on the basis of this definition, that Aristotle agreed with modern thought on the sense of a logical research; but the distinction lies exactly in this word “radically”, as it has been
logic is ontologically compromised\textsuperscript{48} because it wants to be a doctrine strictly linked to reality and always referring to the real foundation of its premises: “When one aims to truth, he has to found his research on premises that express a real inherence”\textsuperscript{49}. 

This doesn't mean that Aristotle didn't know the difference between thought and reality, nor that he couldn't recognize these two fields and keep them distinct\textsuperscript{50}: but, apart from the simple ontological aim that he assumes in questions that we would consider

\textsuperscript{48} See also G. Pasquale, Il principio di non-contraddizione in Aristotele, Bollati Boringhieri, Torino 2008, pp. 25 e 31; M. Malatesta, La logica delle funzioni, Millennium Romae, Roma 2000, pp. 108 and 111: “[...]Lo stagirita assunse personalmente l’impegno ontologico dal momento che richiese la condizione “των ὂντων”[...]). It’s enough to think to the opening of the ninth chapter of De Interpretatione, where the Philosopher expressly distinguishes between beings (and past things) and future things: “ἐπὶ μὲν οὖν τῶν ὂντων καὶ γινομένων ἀνάγκη τὴν κατάφασιν ἢ τὴν ἀπόφασιν ἡλικθῆ ἢ ψευδῆ εἶναι [...]. ἐπὶ δὲ τῶν καθ’ ἑκάστα καὶ μελλόντων ὀυὶ ὧμοιος”. See also An. Pr. 34b5-15 for a good example on the way Aristotle uses physical or metaphysical considerations in logical discussions.

\textsuperscript{49} An. Pr. 81b23-24.

\textsuperscript{50} Cat. 12b5-10: here Aristotle clearly makes a distinction between the field of logic and field of reality, but immediately shows (see also next note) how the logical opposition between affirmation and negation corresponds to the real opposition between the things, to which affirmation and negation are referred (Pasquali, cit., p. 24: “Secondo Aristotle l’opposizione non è solo tra le due proposizioni o espressioni (quella affermativa e quella negativa). Vi è anche contraddizione tra le cose che esse rappresentano”). See also Rhet. 1359b33-34, where Aristotle clarifies that dialectics and rhetoric are about discourses (λόγοι), and not things (πράγματα). See also E. Berti, Dalla dialettica alla filosofia prima, Bompiani, Milano 2004, pp. 72-75 and

670: “La differenza tra il procedimento dialettico e il procedimento fisico, più volte sottolineata da Aristotele mediante la distinzione tra l’indagare “dialetticamente” e l’indagare “fisicamente”, consiste nella differenza tra il cercare spiegazioni di tipo esclusivamente logico-linguistico e il cercare invece spiegazioni di tipo causale. [...] Nulla vieta, tuttavia, che tale scopo [cioè il reperimento di cause] venga raggiunto per mezzo di un procedimento dialet-
ate formal, the passages in which the connection between logic
and reality is explicit are very numerous\textsuperscript{51}.

To conclude this first chapter some summarizing remarks can be
made: Aristotelian logic contains many references to reality, and
keeps with it a steady and necessary link, whose lack can com-
promise its rigor\textsuperscript{52}. Aristotle himself (as we have seen),
often uses philosophical and ontological consideration as a ground for the
solution of logical and dialectical problems: to separate these two
dimensions of the Stagirite's works would be a misleading ana-
chronism\textsuperscript{53}. How can this result help our research?
The PNC is at the same time an axiom of demonstration and a
“transcendental of being-qua-being”\textsuperscript{54}. It concerns propositions
and things, logic predication and real inherence, names and what
they mean.

Ultimately, the treatment of PNC belongs to first philosophy, but
it's also a matter of dialectic, because it's counted as a principle
common to all sciences. The fourth book of \textit{Metaphysics} states,
analyses and defends it as a principle of being; dialectic com-
municates with all sciences because it concerns the common prin-
ciples of all of them. These two fields are strictly linked.

During the study of the features and formulations of the PNC, at-
tention shall be paid to the results of these pages, in order not to
fall into the mistake of “splitting” the principle in two (or three)
different principles, linked only by inclusion/implication relation-

\textsuperscript{51} De Int. 18a34-35: “\textit{εἰ γὰρ πᾶσα κατάφασις ἢ ἀπόφασις ἀληθῆς ἢ γεινής, καὶ ἂπας ἀνάγκη ἢ ὑπάρχειν ἢ μὴ ὑπάρχειν […] Εἰ γὰρ ἀληθῆς εἰπεῖν ὅτι λευκόν ἢ οὐ λευκόν ἢ ὅτι λευκόν ἢ οὐ λευκόν, καὶ εἰ ἔστι λευκόν ἢ οὐ λευκόν, ἀληθῆς ἢ φάναι ἢ ἀσφάλεια}”; Cat. 12b10-16, which follows the passage quoted in the precedent note. See also G. Pasquale, \textit{cit.}, p. 24: “L’impossibilità di pre-
dicare predicati contraddittori del medesimo oggetto equivale all’impossibilità
di affermare e negare una proposizione nel medesimo tempo. Questo ci mostra
l’interdipendenza che esiste tra logica e realtà, nonostante non si possa ridurre
l’una all’altra”.

\textsuperscript{52} See also E. Berti, Contraddizione e dialettica negli antichi e nei moderni,
L’Epos, 1988, Palermo p. 146: “non è vero che la logica di Aristotle fosse pura-
mente formale, in quanto si occupava di proprietà e di nessi reali degli oggetti”.

\textsuperscript{53} See also Düring, \textit{cit.}, p. 66

\textsuperscript{54} E. Berti, \textit{Cit.}, p. 467.
ships. A similar step would create a fracture between the two areas considered in this paragraph, and would link them only at the price to admit the major importance or priority of one of them on the other one.

2. Time has come to face directly the PNC as Aristotle has it stated. He gave various formulations of it, placed in many of his works, and in this chapter I want to analyze them separately and look for their connections. They're principally two: a propositional and a functional formulation. The propositional formulation corresponds to the one, that's usually called “logical formulation” of the PNC (or logical PNC). I preferred to use this name for prudence, because the logical nature of this formulation is to be verified. The functional formulation corresponds to what's usually called “ontological formulation” of PNC (or ontological PNC). I called this formulation “functional” because it's not about propositions, but the attribution of a predicate to a subject. Again, this choice is motivated by prudence. In order to conduct this research in a better way, I'd like to specify what I refer to when I write about “formulations”: they constitute the different ways Aristotle used to deal with the same principle. Their differences allow us to underline some aspects of PNC, or to specify some features of it, but ultimately the difference between these formulations is just apparent: they are just a a more precise or analyzed version of the other.

2.1. Λόγος ἀποφαντικός (as to say ἀπόφασις, “proposition”) is that to which it pertains to be true or false. A simple proposition is a proposition that, through the link (συμπλοκή) between name and verb, is “significant about the inherence or not inherence of something”.

The propositional formulation of PNC negates that two contradictory propositions can be both true. This type corresponds the expression from book Γ: “Let so the strongest notion be this one: that

\[ \text{ἐν ὧ τὸ ἀληθεύειν ἢ ψεύδεσθαι ὑπάρχει} \].

\[ \text{ἔστι δὲν ἡ μὲν ἁπλὴ ἀπόφασις φωνὴ σημαντικὴ περὶ τοῦ εἰ ὑπάρχει εἰ ἢ μὴ ὑπάρχει}. \]

55 De int. 17a5: “ἐν ὦ τὸ ἀληθεύειν ἢ ψεύδεσθαι ὑπάρχει”.
56 ib. 17a23-24: “ἑστὶ δ' ἡ μὲν ἁπλὴ ἀπόφασις φωνὴ σημαντικὴ περὶ τοῦ εἰ ὑπάρχει εἰ ἢ μὴ ὑπάρχει”.

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contradictory statements cannot be true together […] has been shown sufficiently.\(^57\) In order to understand this statement, we have to look at the Aristotelian theory of the opposition between propositions, and to consider shortly what meaning he gave to truth and falseness.

2.1.1. There are four kinds of oppositions: between relatives, between contraries, between privation and possess, and between affirmation and negation.\(^58\) The first three ones pertain to terms, the last one regards propositions.\(^59\) Only the last one, so, is involved in this formulation of PNC. Aristotle, in fact, specifies that only in the opposition between contradictory propositions is it necessary that the one is true and the other one false, and this for two reasons: the first is that they're propositions and not terms. Only propositions can be true or false, because only what is said “according to connection” (κατὰ συμπλοκὴν) has this feature.\(^60\) Because contrary terms and the relationship of possession/privation, also when expressed “according to connection” (as to say, the proposition on the contraries and on possession and privation) are not necessarily one true and the other one false, also if they are opposed in a way similar to contradictory propositions. For example, “Socrates is healthy” and “Socrates is sick” would be both false if Socrates didn't exist.\(^61\) For privation and possession it is the same.\(^62\) Two contradictory propositions, on the contrary, are necessarily always one true and one false, even in the case that the subject they refer to doesn't exist.\(^63\) What are, therefore, contradictory propositions?

\(^57\) *Met.* 1011b12-16: “ὅτι μὲν οὖν βεβαιοτάτη δόξα πασῶν τὸ μὴ εἶναι ἀληθῆς ἀμα τὰς ἀντικειμένας φάσεις […] τοσαῦτα εἰρήσθω”.

\(^58\) *Cat.* 11b17-20: “λέγεται δὲ ἑτερον ἑτέρω ἀντικεῖσθαι τετραχῶς, ἢ ὡς τὰ πρὸς τι, ἢ ὡς τὰ ἐναντίων, ἢ ὡς στέρησις καὶ ἐξίς, ἢ ὡς κατάφασις καὶ ἀπόφασις”.

\(^59\) See also G. Pasquale, *cit.* p. 23.

\(^60\) *Ib.* 13b10-12.

\(^61\) *Ib.* 13b14-18

\(^62\) *Ib.* 13b20-21.

\(^63\) *Ib.* 13b27-29: “ἐπὶ δὲ γε τῆς καταφάσεως καὶ τῆς ἀποφάσεως ἀεὶ, ἐὰν τε ἢ ἐὰν τε μὴ ἢ, τὸ μὲν ἐτερονέστατα ψεῦδος τὸ δὲ ἐτερον ἀληθῆς”.}
Aristotle writes: “let's consider in fact that, among composed propositions these are the ones that oppose each other as contradictory, those that are ordered according to the [verb] to be and not to be”\textsuperscript{64}. For example, the contradictory proposition of “Mario is good” is “Mario is not good”.

But here the difference between contradictory propositions and propositions about contradictory terms has to be specified (considering just individual, not modalized and not quantified propositions). Contradictory terms are, for example, “good” and “not good”. But two propositions that state the contradictories are not contradictory: they need to be one an affirmation, and the other one the only negation possible\textsuperscript{65}.

But the propositions that affirms of a substrate the term, that is contrary to the one affirmed by another proposition, is not a negation, but an affirmation. So, following the indications of the Philosopher, we have four kinds of proposition:

a) X is Y – man is white  
b) X is not Y – man is not white  
c) X is non-Y – man is non-white  
d) X is not non-Y – man is not non-white

i) c) and b) are not two formulations of the same proposition. The first is an affirmation, the other is a negation. a) and c) are propositions concerning contradictory terms. a) and b) are contradictory. c) implies b): in fact if it's true that “not-to-be-white” belongs to a man, then “being white” doesn't belong to him. The contrary, though, is not true: in fact it's possible that what is not white doesn't exist at all, and so that nothing can be predicated of it. Ultimately, the affirmation of negative predicate supposes the existence of the substrate to which it's referred to\textsuperscript{66}. It's slightly in-

\textsuperscript{64} De int. 21a38-39: “εἰ γάρ τῶν συμπλεκομένων αὐτὰ ἀλλήλως ἀντίκεινται αἱ ἀντιφάσεις, ὅσαι κατὰ τὸ εἶναι καὶ μὴ εἶναι τάττονται”. Vedi anche 17a34-35: λέγω δὲ ἀντικεῖσθαι τὴν τοῦ αὐτοῦ κατὰ τοῦ αὐτοῦ”.

\textsuperscript{65} Ib. 17b37: “Φανερὸν δ' ὅτι καὶ μία ἀπόφασις μᾶς καταφάσεως.”

\textsuperscript{66} An pr. 51b25: “the being that is not equal requires a substrate, that is the unequal”. see also M. Zanatta, commentary to Della interpretazione, Rizzoli,
exact what Gianluigi Pasquale states in his book on the Aristotelian PNC: the impossibility to affirm and negate the same proposition at the same time does not correspond to the impossibility to predicate contradictory terms of the same subject, but to the impossibility to predicate and not to predicate the same predicate of the same subject\textsuperscript{67}.

2.1.2. What is left to be determined is what Aristotle means with true and false. Two chapters of the \textit{Metaphysics} are dedicated to this argument, the fourth of book E and the tenth of book \Theta. “Is in truth who thinks separated things to be separated, and linked things to be linked; is in falseness who thinks the things to be different to how they actually are”\textsuperscript{68} Considering the eventual middle term between contradictories, he writes: “This is evident, as we define what true and false is: false is to say that being is not and non-being is; true, on the contrary, is to say that being is and non-being is not”\textsuperscript{69}. “Being”, here, has an equivocal value: it refers in

Milano 2000, p. 276-278; and also \textit{An. Pr.} 51b5-32, where Aristotle rigorously deals with this problem and expressly writes: “On the other hand, when a formulation is consolidated or demolished, there’s a certain difference between thinking that the expression ‘not to be this’ and the expression ‘to be non-this’, and thinking that these two expressions \textit{has different meanings}”. He will solve the question agreeing with the second option, and demonstrating the inherent contradiction of the first.

\textsuperscript{67} G. Pasquale, \textit{cit.}, p. 24: “L’impossibilità di predicare predicati contraddittori del medesimo oggetto corrisponde all’impossibilità di affermare e negare una proposizione nel medesimo tempo”. The critique doesn’t touch what he states about what this implication means for the relationship between logic and reality; it just underlines that in this affirmation can be found a light confusion between impossibility to affirm the contradictories and impossibility to affirm with truth the contradictories (and consequently, the impossibility for the contradictories to be both true).

\textsuperscript{68} \textit{Met.} 1051b3-4: “ἀληθεύει μὲν ὁ τὸ διῃρημένον οἰόμενον διηρήσθαι καὶ τὸ συγκείμενον συγκεῖσθαι, ἐψευσθαι δὲ ὁ ἐναντίως ἔχων ἢ τὰ πράγματα”. A similar definition can be found in book E, 1027b20-22. Here, however, is interesting the not on the concrete things as effective cause of truth and falsity of a proposition.

\textsuperscript{69} \textit{Met.} 1011b25-27: “δῆλον δὲ πρῶτον μὲν ὁρισμένος τι τὸ ἀληθὲς καὶ ψεύδος. Τὸ μὲν γὰρ λέγειν τὸν μὲν εἶναι ἢ τὸ μὴ ὃν εἶναι ψεύδος, τὸ δὲ τὸ ὅν εἶναι καὶ τὸ μὴ ὃν μὴ εἶναι ἀληθὲς”.
fact, not only to the existence of an entity, but also and in particular to the inherence of a determination to a substrate\textsuperscript{70}.

But what is the relation between truth/falseness and things? And so, what's the relation between propositions and their content? Aristotle specifies that true and false are determinations that belong to propositions, and not to things\textsuperscript{71}. On the other hand, there's a strict causal relationship between the truth of propositions and things. He writes that "if, in facts, true is to say that (a thing) is white or not-white, it's necessary for it to be white or not-white; and if it's white or not-white, it was true to affirm or negate it"\textsuperscript{72}. Reading this expression, it might seem that there's a relation of "reciprocal causality" between truth of propositions and actuality of things. But in other passages Aristotle specifies that it's the existence or not-existence of the relation, that the proposition affirms, that makes the last true or false, and not vice versa: "οὐ γὰρ διὰ τὸ ἡμᾶς οἴεσθαι ἀληθῶς σε λευκὸν εἶναι εἰ οὐ λευκός, ἀλλὰ διὰ τὸ σὲ εἶναι λευκόν ἡμεῖς οἱ φάντες τοῦτο ἀληθεύομεν"\textsuperscript{73}. The contradiction between the two passages is just apparent: the logical implication is, in fact, not equal to the cause-effect relation. The existence of facts is cause of the truth of propositions\textsuperscript{72}: the proposition “the lamp is turned on” is true because the lamp is turned on.

\textsuperscript{70} How previously shown, the proposition essentially means the belonging of a predicate to a subject (that is “union” and “disunion”): even where the form concealed it, it could be modified to make this structure explicit. See also De Int 21b9-10: “οὐδὲν γὰρ διαφέρει εἰπεῖν ἄνθρωπον βαδίζειν ἢ ἄνθρωπον βαδίζοντα εἶναι”.

\textsuperscript{71} Met. 10027b25-26: "οὐ γὰρ ἔστι τὸ ψεύδειν καὶ τὸ ἀληθεύειν εἰς τοὺς πραγματικοὺς, οὐδὲ τὸ μόνον αὐθαναλθῆς”.

\textsuperscript{72} De int. 18a39b3: "εἰ γὰρ ἀληθεύειν ὅτι λευκόν ἢ οὐ λευκόν ἔστιν, ἀνάγκη εἶναι λευκόν ἢ οὐ λευκόν, καὶ εἰ ἔστι λευκόν ἢ οὐ λευκόν, ἀληθεύει ἢ ἀσφαλεῖ ἢ ἀποφαντᾶται”.

\textsuperscript{73} Met. 1051b6-9. On this topic Thomas is absolutely clear: “Unde qui putat dividit quod est divisum in rebus, verus est in sua opinione [...] Non enim ideo tu es albus, qua nos vere existimamus te esse album; sed e converso, ideo existimamus te album, quia tu es albus [...] Hoc autem addit ad manifestandum quod supra dixerat, quod verum et falsum est in rebus componi et dividii. Oportet enim veritatem et falsitatem, quae est in oratione vel opinione, reduci ad dispositionem rei sicut ad causam”.

\textsuperscript{74} For “existence of facts” I mean the actual existence of a state of things.
turned on. On the other hand, the truth of a proposition implies the actuality of the fact that is its content: if the proposition “the lamp is turned on” is true, so the lamp is turned on (this is because the truth of a proposition consists precisely of the existence of the fact that it expresses, and is this because of the semantic link that relates affirmation and inherence\(^75\)). It's not a only, one-to-one process, but two different relations\(^76\).

Now can be stated a conclusion, that will be useful in the next chapters: the truth of a proposition is not a logical, but an ontological matter. Logics (Analytics) considers the formal relation between propositions, but their truth must be taken from reality, and for this reason it wants to mean something real. That two propositions cannot be both true together means precisely that, that two contradictory facts cannot be together true.

2.1.3. Between two contradictory propositions there's a relation that is not between other propositions: contrary propositions can be, as has been seen, both false; other propositions can also be both true; but in the case of two contradictory propositions, it's necessary for one of them to be true, and the other false\(^77\). It can be shown that this affirmation can be reduced to the propositional formulation of PNC. We need two elements:

\[\text{a) The 13}^{\text{th}} \text{ chapter of } \text{De interpretatione} \text{ shows the implication between the concept of possible, contingent, necessary and impossible. Here Aristotle states in a clear way that it is “necessary} \]

\(^75\) See note 1. Moreover, let's consider Aristotle's statement that “discourses are true in a similar way, to the way facts are” “ὁμοίως οἱ λόγοι ἐν τῇ ληθείᾳ τὰ πράγματα”.

\(^76\) Aristotle's specifying on the causal link between truth and facts shows that he knew this distinction. On this topic Lukasiewicz writes that for Aristotle there was a purely logical equivalence, and not real, between expressions. But it will be shown that he made a confusion between two different things: what he calls logical PNC (it's impossible that two contradictory affirmations are together true) and what he talks about when he shows this logical equivalence (negating or affirming a thing).

\(^77\) See also i.e. De int. 18a28-35.
that something is” is equivalent to “impossible that something is not”\textsuperscript{78}.

b) The truth of affirmation implies the falseness of the negation, and vice versa. As to say, between two contradictory propositions there’s a link, for which the truth or falseness of the first implies, respectively, the falseness or the truth of the second\textsuperscript{79}.

Well, let’s now take as an example two contradictory propositions like “Mario is good” (p) and “Mario isn’t good” (Np): the statement “it’s necessary for two contradictory proposition, to be one true and the other false” is equivalent to the statement “it’s impossible for two contradictory propositions to be not one true and the other false”. That means that it's impossible for them to be both true or both false. The first case is precisely what the PNC negates.

For the second case: if we can demonstrate that both the formulations negate that two contradictory propositions can be both false, the equivalence between PNC and the given principle will be demonstrated.

The truth of p implies the falseness of Np; the truth of Np implies the falsity of p. Using a similar method as the one used by Aristotle in some passages of Analytics, it's evident that the falseness of the two proposition implies the truth of both propositions.

\[
\begin{align*}
F_Np & \leftrightarrow V_p \\
F_p & \leftrightarrow V_Np \\
(F_p \land F_Np) & \leftrightarrow (V_Np \land V_p) \tag{80}
\end{align*}
\]

\textsuperscript{78} De int. 22a21-22

\textsuperscript{79} I.e. Met. 1008a34-36: here Aristotle understands that establishing this kind of link between contradictory propositions already means stating the PNC, and consequently falling in a petitio principii.

\textsuperscript{80} I explain the usage of symbols: the letters written in \textit{algerian} are symbols of the metalanguage that are used to indicate the truth (V) and falseness (F) of a proposition. “N” is the symbol of negation, and consequently “Np” is for “non p”. The symbol “ $\leftrightarrow$ ” indicates the logical equivalence, and means “if and only if”; finally, the symbol “$\land$” means “et”, and indicates the logical product.
There's also a simpler way to demonstrate this: the truth of affirmation is the falseness of negation, and the truth of negation is the falseness of affirmation. Now, the PNC states that contradictory propositions cannot both be true: so neither the two particular contradictory propositions that are the negations of the two given statements can be true. So, the impossibility for two contradictory propositions to be both true is equivalent to the impossibility for them to be both false. So it's necessary for them to be one true and the other one false.

What has been shown is the equivalence between the PNC and the so-called “law of the excluded middle”: this is often presented as a “consequence” of the PNC, or as corollary. Nevertheless, these two expositions mean exactly the same thing, that is that two contradictory propositions cannot be both true or both false: what changes is simply the negative aspect of the formulation, in one case “affirmative”, in the other one “negative”.

In summary: 1) The PNC, affirming that two contradictory propositions cannot be both true, means automatically that they cannot be both false, and so that necessarily one of them is true, and the other false. 2) The Law of the excluded middle (LEM), affirming that of two contradictory propositions one is necessarily true and the other false, negates the chance for them to be both true and both false. 3) In the end, it can be said that they are not two different principles but two different ways to express the same principle.

2.1.4. Let's briefly summarize what has now been gleaned about the PNC:

1) The propositional formulation of PNC states that two contradictory propositions (that is affirmation and negation) cannot be both true together.

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81 This expression is legitimated by Met. 1012b9-10: \(\varepsiloni\ \deltae\ \mu\theta\varepsilon\ \alpha\lambda\lambda\o\i\i\ \theta\eta\varepsilon\ \phi\varepsilon\alpha\i\alpha\ \varepsiloni\ \varepsilon\pi\varepsilon\\delta\iota\nu\varepsilon \delta\tau\iota\nu\iota\i\). 82 See also E. Berti, *Contraddizione e dialettica*, cit. p. 115.
2) That means that they also can't be both false. The PNC, so, states that of two contradictory propositions one has to be true, and the other false.

3) Because the truth of affirmations is the existence of the facts, the PNC states, in the end, that two contradictory facts cannot exist together; that means, that a fact cannot exist and not-exist at the same time and in the same meaning.

2.2. The functional formulation of PNC is so called because it regards the relation between subject and predicate, that is substrate and attribute. It's the most known and complex formulation of the principle, also because it's the one stated and treated in the book Γ of Metaphysics.

It reads as such: “τὸ γάρ αὐτὸ ἅμα ὑπάρχειν τε καὶ μὴ ὑπάρχειν ἀδύνατον τῷ αὐτῷ καὶ κατὰ τὸ αὐτὸ”\(^{83}\). I give three different translations:

1) “E’ impossibile che la stessa cosa, ad un tempo, appartenga e non appartenga a una medesima cosa, secondo lo stesso rispetto” (tr. Reale) [It's impossible for the same thing, at one time, to belong and not to belong to the same thing, according to the same respect]\(^{84}\)

2) “E’ impossibile che la stessa cosa appartenga e contemporaneamente non appartenga alla stessa cosa e sotto lo stesso aspetto” (tr. Berti) [It's impossible that the same thing belongs and at the same time doesn't belong to the same thing and according to the same aspect]\(^{85}\)

3) “E’ impossibile che la stessa cosa convenga e insieme non convenga ad una stessa cosa e per il medesimo rispetto” (trad. Severino) [It's impossible for the same thing to be proper and together not to be proper of the same thing according to the same respect] \(^{86}\)

\(^{83}\) Met. 1005b19-20

\(^{84}\) Aristotele, Metafisica. Vita e pensiero, 1995, pp. 143-145.

\(^{85}\) E. Berti, Contraddizione e dialettica negli antichi e nei moderni, L’epos, Milano 1988, p. 103.

\(^{86}\) E. Severino, Aristotele “il principio di non contraddizione”, in Fondamento della contraddizione, Adelphi, Milano 2005, p. 258
I'll use Severino's translation because I think it's more literal (and so a little more prudent). The best way to proceed seems to be this: to analyse the formulation word for word, because each of them reveals something of the content and of the meaning of the so-formulated PNC.

2.2.1. Ἁδύνατον. The meaning of impossibility is expressed by the philosopher of Stageira in book Δ: “impossible is whose contrary is necessarily true”\(^{87}\). The notion of impossibility is related to the notion of necessity and true/false. It has been show how in *De interpretatione* Aristotle established the relation between necessity and impossibility\(^{88}\); it can be so said that impossible is that for which it's necessary not to be. The impossible is what is necessarily false (and consequently whose contrary/contradictory\(^{89}\) is necessarily true). Aristotle uses as an example the commensurability of the diagonal with the side of a square: it's impossible, because its incommensurability will be forever true. It's easy to see, how that which justifies this meaning of impossibility is the PNC itself: the diagonal can't be commensurable to the side of the square because its incommensurability is always true, if it were it would be commensurable and not commensurable at the same time. According to this example, the impossible is only about what is always: but as E. Severino correctly remarked\(^{90}\) that's not true. It's evident, when one references the ninth chapter of *De interpretatione*: “it is so necessary to be for what is, when it is, and not to be for what is not, when it's not”\(^{91}\). Impossible is, in this case, simply that something that is is not when it is (and so it's impossible for it to be and

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\(^{87}\) Met. 1019b23-24: “ἀδύνατον μὲν οὗ τὸ ἐναντίον ἐξ ἀνάγχης ἀληθὲς”.

\(^{88}\) See also note 23.

\(^{89}\) Aristotle does not always rigorously distinguish between these two terms.

\(^{90}\) Fondamento della contraddizione, p. 32: “Ed è necessario che “falso” sia sinonimo di “impossibile”, perché, anche per le affermazioni le cui contraddittorie non sono in sé autocontraddittorie, cioè impossibili, va detto che, sin tanto che (ossia quando) a S conviene P, è impossibile che P non gli convenga”.

\(^{91}\) *De int*. 19a22-23: “τὸ μὲν οὖν εἶναι τὸ ὃν ἐστιν ἢ, καὶ τὸ μὴ ὃν μὴ εἶναι ἀποφάσαται μὴ ἢ, ἀνάγκη”.
not to be), and that something that is not is when it is not (the same). So, we have again the PNC\(^{92}\), and a new meaning of impossibility extended to changing things as well as to eternal ones. So it is not only impossible for the same predicate to belong and not to belong to the same under the same aspect, but impossible is precisely this: the PNC is a definition of impossible; impossible is the contradictory\(^{93}\).

2.2.2.  (){This term is translated with “to belong” or “to be proper of”. Łukasiewicz calls it “inherence”, meaning with this the owning of an attribute by a subject.\(^{94}\) The PNC states, in this formulation, that the same feature cannot together be proper and not to be proper of the same substrate. There are some points to clarify on this element, that for obvious reasons compose a crucial phase of the exposition of PNC:

1) If the \( \gamma \nu \alpha \zeta \gamma e \iota \) is of just one type, or if it can be considered under different aspects, according to the case.

2) What's the relation between \( \gamma \nu \alpha \zeta \gamma e \iota \) and \( k a t e g o r e i \). In this way will be possible to think about the meaning of this formulation, if is or is not solely ontological.

On the first question: Aristotle distinguishes evidently two kinds of predicators, or inherence between subject and attribute. This is implicit in his ontology, based on the difference between \( \omicron \upsilon \zeta \iota \alpha \) and accident \( \sigma \mu \beta \varepsilon \varepsilon \kappa \omicron \). Proceeding with the defense of the principle, and so writing of the inherence of a feature to a subject, the Stagirian makes a difference between “meaning a thing” and

\(^{92}\) It is necessary here to note on the link between PNC and principle of double negation. This link will be treated in the next chapters, with the intent to show that this is, again, a different way to formulate the same principle.

\(^{93}\) Of the same opinion is E. Severino, that takes it from the Aristotelian definition of true and false (Fondamento della contraddizione, pp. 32-33), in a way that identifies this definition with the formulation of PNC: “Dicendo che è impossibile che allo stesso convenga e insieme non convenga lo stesso, Aristote dice che l’impossibile è che allo stesso convenga e non convenga lo stesso”.

\(^{94}\) J. Łukasiewicz, Del principio di contraddizione in Aristotele, Quodlibet, Macerata 2003, p. 19
“meaning the attribute of a determined thing”\textsuperscript{95}. We've so two different cases: in the first one it's said, of a thing, what it essentially is, in the second instead it's said just a non essential, accidental feature of a thing\textsuperscript{96}. This follows precisely another distinction by Aristotles, traced in the \textit{Categories}, between what is said of a subject, and what is said in a subject\textsuperscript{97}: the first are definitions, and so they express what the subject is; the seconds are not-definitory predications, that can be more or less accidental. Enrico Berti underlined that the difference is between an intra-generic way of predication and an extra-generic one: what is said of a subject it to it as the second substance to the first substance\textsuperscript{98}. What is said in a subject is to it, normally, as the accident to the first substance, and is the predication of other categories to \textit{ousia}\textsuperscript{99}.

In this last case it's an inessential predication, because, being the predicate not included in the essential definition of the subject, the subject remains the same also without the predicate (which doesn't happen in the first case, and so makes it immediately contradictory to not affirm the definition of the subject).

\textsuperscript{95} Met. 1006b15-17: “οὐ γὰρ τοῦτο ἀξιοῦμεν τὸ ἐν σημαίνειν, τὸ καθ’ ἐνὸς”.
\textsuperscript{96} See also G. Pasquale, \textit{cit.}, p. 25
\textsuperscript{97} Cat. 1a20-22: “τῶν ὅντων τὰ μὲν καθ’ ὑποκειμένου τινὸς λέγεται, ἐν ὑποκειμένῳ δὲ οὐδενί ἔστιν, […] τὰ δὲ ἐν ὑποκειμένῳ μὲν ἔστιν, καθ’ ὑποκειμένου δὲ οὐδενὸς λέγεται”. See also Berti, \textit{cit.}, p. 259: “La predicazione può avvenire sia tra termini compresi nella stessa colonna, quando ad esempio un genere si predica di una specie o di un individuo che costituiscono un suo caso particolare, sia fra termini appartenenti a colonne diverse, quando ad esempio un genere o una specie o un individuo di una colonna si predicano rispettivamente di un genere, di una specie o di un individuo di un'altra. Aristotle ha distinto il primo tipo di predicazione dal secondo, affermando che nel primo il predicato è propriamente un predicato, cioè qualcosa che è detto di un soggetto, mentre nel secondo il predicato non è propriamente un predicato, ma è qualcosa che è presente, o inerente, in un soggetto”. In Berti's opinion this allows the difference between ononymy and synonymy.
\textsuperscript{98} Cat. 2a11-25
\textsuperscript{99} Aristotle seems to specify (\textit{Cat. 1A24-25}) that what is said in a subject is not separable from what it is in: but this, in my opinion, only in the meaning that such a predicate can't be considered separately from the subject it refers to. Further Aristotle makes the example of white, predicating it of a body. Obviously he couldn't think that it was impossible, for a body, to change colour.
We have two cases: a) first substance-second substance predication b) substance-other categories predication. The first is, as we saw, an essential relationship; the second is an accidental relationship\textsuperscript{100}. The PNC involves both.

a) The greatest share of the examples that Aristotle uses in order to defend the PNC in the book \(\Gamma\) of \textit{Metaphysics} is of this kind. This also because what particularly worries the Stagirite is preserving the distinction between \(\text{όσια}\) and accident, subject of predication and predicate. Negating the relation between something and its definition is immediately contradictory.

One thing has to be clarified: the essential predication is not the substantial predication, that means that it doesn't regard only substances. There's a difference, so, between substance and essence: the first one is something meaningful, the second is a meaning, something that defines and determines. Aristotle writes that of the things that are said of a subject, also the definition of them is said of that subject. The same is not in the case of the definition of things that are said in a subject\textsuperscript{101}. But this doesn't mean that of the things that are said in a subject there's no definition! The PNC preserves the essence of substance as much as the essence of accident, and both are non accidental\textsuperscript{102}.

\textsuperscript{100} G. Pasquale, \textit{cit.}, p.25: "Aristotle, nel suo libro delle Categorie, distingue dieci categorie: sostanza, qualità, quantità, relazione, luogo, situazione, tempo, movimento ecc. La prima delle categorie, la sostanza, dice qualcosa di essenziale del soggetto, mentre le altre nove categorie dicono qualcosa di accidentale". It will be shown how this is valid only for the essential definition of substance.

\textsuperscript{101} Cat. 2a19-35: \textit{"φανερὸν δὲ ἐκ τῶν εἰρημένων ὅτι τῶν καθ’ ὑποκειμένων ἱερημένων ἔναγκαιὸν καὶ τὸν λόγον κατηγορεῖσθαι τοῦ ὑποκειμένου (...) τῶν δὲ ἐν ὑποκειμένῳ ὄντων ἐπί μὲν τῶν πλείστων ὄντων τὸν λόγον κατηγορεῖσθαι"}.

\textsuperscript{102} E. Severino is, on this topic, really clear: \textit{"Si badi: dire che la negazione del p.d.n.c. viene \textquoteleft a sopprimere la sostanza e l’essenza\textquoteleft non significa che ciò che viene ad essere soppresso è un certo tipo di oggetti, una certa parte della realtà fatta esclusione dell’altra (ad esempio, che vengono soppressi oggetti come \textquoteleft uomo\textquoteleft, \textquoteleft animale\textquoteleft, ma non oggetti come \textquoteleft bianco\textquoteleft, \textquoteleft pesante\textquoteright). Infatti, è vero che oggetti come \textquoteleft bianco\textquoteright, \textquoteleft pesante\textquoteright sono accidenti rispetto a questa o quest’altra realtà corporea, ma essi stessi presentano un’essenza, un significato...\"}. 24
b) The negation of the relation between substance and accident is not immediately contradictory: it's contradictory, nevertheless, to affirm and to negate together this relation. So, the PNC regards also the accidental predication\textsuperscript{103}, that means the extra-generic, and this for various reasons: the first one is that Aristotle expressly makes a parallel between ἀπόφανσις and ὑπάρχειν: The first means the belonging or not belonging of something to something\textsuperscript{104}. But consisting the proposition in predication, according to a subject, of any of the ten categories (the “predicables”, precisely), it's self-evident that the meaning of ὑπάρχειν is not limited to the essential, or intra-generic relation. Moreover, Aristotle expressly formulates examples based on the relation between substance and accident\textsuperscript{105}: falling in a well can't be at the same time good and not-good, or the same object can't be at the same time (and under the same aspect) sweet and not-sweet\textsuperscript{106}.

Concluding the remarks on the first question, the PNC concerns: 
a) the substance-essence relation 
b) the accident-essence relation 
c) the substance – accident relation. For this reason the thesis of Pasquale, according to which the aim of PNC is to preserve the unity of substance and of accident, is incomplete\textsuperscript{107}.

On the second question: from what stated already proceeds the complete correspondence between predication and inherence (in
the meaning of κατηγορεῖν and ὑπάρχειν). It has been seen how, in
the propositional formulation of PNC, the truth of the proposition
consisted exactly in the existence of the fact that it expressed. In
the functional formulation, instead, the belonging of something to
a subject consists precisely in its being predicated (with truth) of
that subject. Aristotle, let's remark that again, knew the difference
between terms and things, but didn't lose the absolute correspond-
ence of these two worlds.

Various aristotelic passages witness this link: at the beginning of
the Categories Aristotle writes expressly that the things that are
said of a subject are ὅντα, real entities and not just words. The
word itself, that Aristotle uses to indicate the subject of proposi-
tion (ὑποκείμενον) is used in both meanings, logical (in the Cat-
egories) and ontological (in the Metaphysics, as substrate). Other
passages show the deep entwining of the two dimensions: “And,
because it's impossible for the contradictories, referred to the same
thing, to be together true, it's evident that neither the contraries can
be together in the same object.” Here Aristotle shows the abso-
lute correspondence between ἀληθεύεσθαι and ὑπάρχειν. When
Aristotle writes that “it's impossible for the contradictories to be
predicated together”, he means precisely that it's impossible for
two contradictory attributes to be predicated of the same substrate.

2.2.3. Αὐτό. Looking for the meaning of this term means also to
discover, ultimately, what are the possible subjects of the predica-
tion and the possible predicates on which the PNC has effect. It
has been seen how the PNC has effect on all the entities indiffer-

108 Cat. 1a20: “τῶν ἄντων, τὰ μὲν καθ ὑποκείμενον ὥς ἔλεγεται”.
109 Talking about the difference between οὐσία and accident, Aristotle writes:
“But if all things are said as accident, it will be nothing that is first subject of
accidents, and accident always is a predicate (κατηγοριαν) of a subject”. In a
sentence like this it's really difficult to understand if Aristotle is talking about
terms or real attributes/beings. In fact, these two realities are so strictly linked,
that the question is not important.
110 Met. 1011b17-20: “ἐπει δ ἄδονατον τὴν ἀντίφασιν ἀμα ἀληθεύεσθαι κατά τοῦ
αὐτοῦ, φανερὸν ὅτι οὐδὲ τάναντα ἀμα ὑπάρχειν ἐνδέχεται τῷ αὐτῷ”.
111 Met. 1007b18: “ἀδόνατον ἀμα κατηγορεῖσθαι εἰς ἀντιφάσις”.
ently. A great part of the results can be extracted from the precedent paragraph:

1) The subject of the PNC can be:
   a. A οὐσία, first or second
   b. An accident (meant in an essential sense)
   c. A definition
   d. A meaningful term

2) The predicate of PNC can be:
   a. A οὐσία, first or second (in the first case we have an identity, in the second case we have the predication of a subject).
   b. An accident (also meant as a simple affection of the οὐσία, as to say as a predication in a subject)
   c. A definition

The PNC is principle of all beings: it refers to what is, and not to what is not. But we've seen (supra, p.4) that “being” is an equivocal term, and that it can be referred to οὐσία as to its affection and negation, and that of non-being itself is said, that it is; for this reason is valid what already stated – that of two contradictory propositions one is necessarily true even in the case of a non-existent subject. The king of France can be bald or not be bald, but not both or none of the two. If the king of France doesn't exist, of him cannot be predicated the being-bald. Again, the PNC in its strongest formulation is not about the inherence of contradictory predicates, but the attribution or not of the same predicate. Here's the possible cases in which the PNC has effect:

112 G. Pasquale analyses very well the difference between opposition between contraries and opposition between contradictories. But then he states that “il soggetto che il principio protegge dalla contraddizione è un soggetto esistente” (cit. p.25). But it's evident that if the PNC was not valid also for the non-existing subject, there wouldn't be the chance to distinguish the contrary proposition from the contradictory ones: the first ones, in fact, could be both false because the subject doesn't exist, and so the second ones, because the PNC is about beings, but beings analogically considered, because of any thing it can be said, in a certain meaning, that it is: for any of these cases the PNC will assume a different value, but will not lose is strength.

113 It can be already remarked that some of these cases can bring the PNC to a
1) **Oũσια – οὐσία**: It's impossible for Socrates to be and not to be Socrates; it's impossible for the man to be and not to be a man; it's impossible for Socrates to be and not to be a man.

2) **Oũσια – accident**: It's impossible for the man to be and not to be good, it's impossible for Socrates to be and not to be good.

3) **Oũσια – definition**: It's impossible for the man to be and not to be a rational animal.

4) **Accident – accident**: It's impossible for the white to be and not to be white (the white, not just white)

5) **Accident – definition**: It's impossible for the white to be and not to be the brightest colour.

6) **Definition – definition**: It's impossible for the definition “rational animal” to be and not to be the definition “rational animal”.

7) **Term – term**: It's impossible for the term “man” to be and not to be the term “man”.

8) **Term – definition**: It's impossible for the term “man” to mean and not to mean “rational animal”.

2.2.4. **Ἄμα**. The meaning of this word has a very important role in the formulation of the PNC: the temporal meaning of the principle. As can be seen in the beginning of the paragraph 2, the usual translation of the principle expresses ἄμα as “at the same time”. That means to reduce the sense of the term to a temporal meaning, and to link the PNC to time. But what does “ἄμα” for Aristotle mean?

The answer is expressly given in the *Categories*, in the post-praedicamenta section: “And so, are simultaneous for nature all those things that are correlative according to the consequence of existing, but that in no way one is cause of the other's existence; and those that from the same genre can be divided in correspond-

formulation of the principle of identity. This one is considered as not formulated by Aristotle, and this will be discussed. For now, let's remark that the statement by Aristotle that “even of non-being is said that it is” (*Met.* 1003b10-11) is a demonstration that what we call principle of identity is extended to a dimension of being that is greater of the one of οὐσία.
ence one with another. In an absolute meaning, are said simultaneous those things, whose generation happens in the same time\textsuperscript{114}. In an absolute meaning, therefore, the word has a temporal validity. Nevertheless, it refers also to things that are simply correlative, without introducing any temporal nexus. It can be stated, ultimately, that if it's true that the PNC negates the possibility for the same predicate to belong and not to belong to the same subject at the same time, it's also true that in a world without time the principle would not only keep it's value, but would have it in an even stronger way.

The meaning of “simultaneously” to which Aristotle refers is mostly temporal, but not only\textsuperscript{115}: it's sufficient to thing to logical or mathematical propositions, in which it's impossible to use the concept of time (for Aristotle it's a physical concept).

2.2.5. Κατά. Ingemar Düring underlined the importance of the Aristotelian use of “as” (qua)\textsuperscript{116}, probably referring to the “ الحي".

\textsuperscript{114} Cat. 15a8-12: “계약 오니ν τη φίλον λέγεται ὅσα ἀντιστρέφει μὲν κατὰ τὴν ἐναί ἀκαλούθησιν, μηδὲν δὲ αἵτων τὸ ἐπερον τῷ ἑπερον τὸ ἑναι ἕστιν, καί τὰ ἐκ τοῦ αὐτοῦ γένος ἀντιστρέφει ἀλλήλοις ἀμαίνοις ἀμαίνοις ἀμαίνοις ἀμαίνοις ἀμαίνοις ἀμαίνοις ἀμαίνοις ἀμαίνοις ἀμαίνοις ἀμαίνοις ἀμαίνοις ἀμαίνοις ἀμαίνοις ἀμαί

\textsuperscript{115} See also Pasquale, cit. p. 33: “Tempo e spazio sono dimensioni entro le quali rappresentiamo il divenire e la molteplicità dell’essere. Tuttavia, esse non sono affatto essenziali per il concetto metafisico del divenire, perché la scienza dell’essere in quanto essere si occupa della differenza tra esseri nel loro modo di essere. Nella formulazione del PNC, il concetto di tempo non è, dunque, una parte essenziale del principio”. Severino (cit. 258) excludes the particle from the group of the “meaningful” elements of the principle, and includes the temporal value of the PNC in the notion of “respect”. Here ἅμα probably expresses a generic sense of simultaneity, where the expression “κατα τὸ αἴτω” takes also the temporal meaning of formulation. On the other hand, there are many places in Aristotle’s work where the aristotelian attention to the temporal element in the determination of the PNC is explicit (maybe following what Plato, i.e. in Symposium, had affirmed). I.e. see also Met. 1061b36: “καθ’ ἑνα καὶ αἰτῶν χρόνον”. In the end, let’s specify that the meaning of Severino’s statement about the dependence of the PNC from the temporal factor doesn’t regard the question of ἅμα, but a theoretical disagreement between Severino and Aristotle that is beyond the scope this essay.

\textsuperscript{116} I. Düring, cit., p. 28: “l’espressione “in quanto”, raramente usata da Platone, fu da lui sviluppata fino a diventare uno strumento che gli dava la possib-
particle, frequently used by the Philosopher. The link between κατά and this word (and the identity of their meanings) is evident in the first chapter of the book Γ of *Metaphysics*: “there iss a science that studies the being-qua (ἵ)-being and the properties that belong to it as (κατά) being”;“necessarily these elements weren't elements of the being as accident (κατὰ συμβεβηκός), but as (ἵ) being”.

With this expression Aristotle wants to specify, against possible sophistical attacks, that it's not enough, for the same attribute, to belong and not to belong to the same thing; it has to belong and not to belong to the same thing, from the same standpoint, in the same case, in the same meaning and considering the same aspect of the mentioned thing: three is greater than two and smaller than four, but it's not greater and smaller than four; a cake is good because it's tasty and not good because it's not virtuous, but it's not virtuous and not virtuous, or tasty and not-tasty. As witnessed from what follows the formulation of the principle119, this expression is a defense against the λογικά δυσχερεία, the linguistic difficulties that can compromise a correct interpretation of the principle.

2.3. The formulations of the PNC that we've considered until now have the same validity: which is to say that they are different ways to formulate the same principle. They're not one the consequence of the other, and neither one a particular case of the other. Moreover, they're two strict formulations that express the principle in the totality of its meaning and its implications.

There are, nevertheless, other Aristotelian affirmations that follow from the PNC, or that can be considered particular cases of it.

117 Met. 1003a20-21.
118 Ib. 1003a30-3.
119 “καὶ ὅσα ἄλλα προσδιορισαίμεθ’ ἢν, ἐστι προσδιορισμένα πρός τὶς λογικάς δυσχερείας”. 
They're not as strict as the ones considered before, they don't have all the consequences that the first have, and ultimately they say less that what the two strict formulations say. Because Aristotle seems to use them as formulations of the PNC, they could be called its “weak” formulations. Aristotle uses them where this version of the βεβαιοτάτη ἀρχή is sufficient to demonstrate what he states.

The principal of these formulations is the following: “it's impossible for the contraries to belong to the same subject”. It can be found, for example, in Met. 1005b26-27: "μή ἐνδέχεται ἄμα ὑπάρχειν τῷ αὐτῷ ταναντία (προσδιωρίσθω δ' ἧμιν καὶ ταύτη τῇ προστάσει τὰ εἴσωθοτα)" [It's not possible for the contraries to belong together to the same (and let's add to this premise the usual specifications)]\textsuperscript{120}.

In order to analyze completely this formulation, two things must be done: a) we must define contrariety and the other kinds of oppositions that in the precedent paragraphs we didn't consider b) we must understand what kind of link there is between the PNC in its strict formulation and this kind of statement.

a) In the paragraph 2.1.1\textsuperscript{121} we considered what Aristotle states about the contradictory opposition of affirmation and negation, the only one regarding propositions; now we have to consider the oppositions between terms, that for Aristotle can be of four kinds:

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\item \textsuperscript{120} See also anche Met. 1019a25-26. E. Severino takes this as another formulation of the PNC (Fondamento della contraddizione, p. 259). This thesis is right only as long as it's specified that it's a less strong and strict formulation of the principle: because, in fact, PNC and law of excluded middle are two different ways to express the same principle, it's evident that this formulation, being not at the same time a formulation of the law, is not as complete as the precedent. As it will be said, that two contraries can't belong to the same subject doesn't imply that one of them must necessarily belong to it.
\item \textsuperscript{121} See also pp. 1-2
\end{itemize}
\end{footnotesize}
oppositions between relatives, contraries, possession and privation, and contradictories.

The relative opposition is between subjects that are correlated, and that can be defined one through the other: for example, the father is father of a son; the son is son of a father; the great is great when compared to small, and the small is small when compared to the great. As G. Pasquale correctly remarked in his essay, the opposition between relatives is not about the PNC, because it's about two subjects, not one single term.

The opposition between contraries is of two kinds: there's contraries with a middle between them, and contraries with no middle. Contrariety is a perfect difference, as it is a difference between extremes, that don't have anything beyond them. It's precisely, the opposition between those things, that are most different.

\[\text{Cat. 11b17-18: "λέγεται δὲ ἔτερον ἐτέρῳ ἀντικεῖσθαι τετραχῶς, ἢ ὡς τὰ πρός τι, ἢ ὡς τὰ ἐναντία, ἢ ὡς στέρησι καὶ ἐξῆς". There's not a word on contradictories, because it's substituted by the treatise of affirmation and negation. See also Met. 1057a36-37.}\]

\[\text{On this topic Aristotle shortly writes in book Δ and particularly I of Metaphysics.}\]

\[\text{Cat. 11b32-35:"ὅσα οὖν ἀντίκειται ὡς τὰ πρός τι αὐτὰ ἢ ὡς ὡς τῶν ἀντικειμένων ἢ ὡς πρὸς ἄλληλα λέγεται: τὰ δὲ ὡς τὰ ἐναντία, αὐτὰ μὲν ἢ ὡς ὡς τῶν ἀντικειμένων πρὸς ἄλληλα λέγεται, ἐναντία μὲντοι ἀλλήλων λέγεται".}\]

\[\text{G. Pasquale, cit. p. 19:"Il PNC definisce il soggetto in relazione a se stesso e afferma del soggetto che è impossibile predicare predicati contraddittori nella medesima relazione e nel medesimo tempo. Invece, l’opposizione di termini relativi rientra nella relazione reciproca che esiste tra due soggetti, per esempio tra padre e figlio. Quindi, l’opposizione dei termini relativi è un’opposizione sulla quale il PNC non influenza né obietta".}\]

\[\text{Ultimately, it's not contradictory for the same man to be at the same time father and son. This doesn't exclude that it's contradictory for the same man to be and not to be father, or to be and not to be son.}\]

\[\text{Cat. 11b38-12a1:"ὅσα δὲ τῶν ἐναντίων ταυτά ἐστιν ὡς ὡς ὧς πάροικος γίγνεσθαι ἢ ὡς καθορίσται ἀναγκαῖον αὐτῶν ἑπάρχειν, τούτων οὐδὲν ἐστιν ἀνά μέσον. Ἡν δὲ γε μὴ ἀναγκαῖον ἑπάρχειν, τούτων ἐστιν τὰ ἀνά μέσον πάντως".}\]

\[\text{"Διαφορά τελέος".}\]
in the same genre\textsuperscript{129}: typical examples are white and black, healthy and sick, good and evil, pretty and ugly.

The opposition between possession and privation is not as formal as the precedent, and implies more physical and natural considerations. Differently from the relatives, privation and possession are said of the same thing, and not one of the other\textsuperscript{130}.

That's how Aristotle defines the two terms: “of all of the things that are able to possess anything, we say that it's deprived when that that should be in something by nature, and that at that time by that something is by nature possessed, doesn't exist in any way”\textsuperscript{131}. It is, therefore, a relation that regards a subject that, by nature, should be able to possess the given determination. The link between privation, contrariety and contradiction is specified by Aristotle in the tenth book of \textit{Metaphysics}: privation is a certain contradiction, but doesn't exclude absolutely the middle as the last. It's necessary – says Aristotle – for the same thing to be equal and not equal, but not equal and unequal (because only what can be unequal can also admit equality)\textsuperscript{132}.

\begin{itemize}
  \item \textsuperscript{129} \textit{Met.} 1018a27-28: “τὰ πλεῖστον διαφέροντα τῶν ἐν τῷ αὐτῷ γένει.”
  \item \textsuperscript{130} \textit{Cat.} 12a26: “Στέρησις δὲ καὶ ἕξις λέγεται μὲν περὶ ταῦτα τι”. \textit{Ivi}, 12b16-18: “ὅτι δὲ ἡ στέρησις καὶ ἡ ἕξις οὐκ ἀντίκειται ὡς τὰ πρὸς τι, φανερῶν: οὐ γὰρ λέγεται αὐτῷ ὅπερ ἄστη τοῦ ἀντικειμένου”.
  \item \textsuperscript{131} \textit{Cat.} 12a29-31.
  \item \textsuperscript{132} \textit{Met.} 1055b3-11. Pasquale (cit. p. 23) writes on the topic: “E' chiaro, quindi, che l'opposizione di possesso e privazione non è la stessa che fra termini contraddittori. Nell'opposizione di possesso e privazione, il soggetto, nonostante manchi di qualcosa che gli appartiene per natura, rimane invece sempre essenzialmente lo stesso e conserva la sua identità. Invece, i predicati contraddittori influenzano l'esistenza o la non-esistenza del soggetto. Se il soggetto è essenzialmente A non può essere essenzialmente non-A”. This consideration falls into an essentialist interpretation of the PNC, according to which this would regard only essential predication, and not the accidental one. A little further Aristotle writes that every contrariety is a privation, but not every privation is a contrariety, because there can be contrary opposition only between extremes, between which there's no movement. Here it's evident that the Philosopher (as he specifies) is talking about contraries, contradictories and privation in a certain meaning: it's already possible to see that if every contrariety is a privation, and every privation is a contradiction, then every contrariety is a contradiction, even if contrariety and contradiction are not the same thing.
\end{itemize}
Aristotle writes about contradictory words in the book A of *Metaphysics*, but briefly and not very strictly (ἀντίφασις is, for the Philosopher, strictly the opposition between affirmation and negation). A good definition of this opposition between terms can be taken from the confrontation with the contraries: unlike these ones, the contradictory terms can't have a middle between them. Being one the simple negation of the other, they both include the whole universe: every being necessarily is x or is not x, without exceptions[^133]. If the contraries belong together to the same genre, the contradictory instead belong to different genres, representing an absolute difference, without any possible point in common: from here the idea that movement is between contraries, not contradictories[^134].

What mostly pertains to this essay is that the contraries are particular contradictories: Aristotle says so expressly in various passages.

In the first passage, taken from the fourteenth chapter of *De interpretazione* (23b7-27), Aristotle writes about contrariety between opinions. What's relevant about the problem of the link between contraries and contradictories is this passage: “If good is both good and not bad, and the first thing is as itself, the second for accident (because it happens to it, not to be bad), and of every thing is truer the opinion relative to the itself, such an opinion is also falser, if it's also truer [...] The opinion that the good is bad is complex, be-

[^133]: *Met.* 1055b1-3: “ἀντιφάσεως δὲ μηδὲν ἐστι μεταξὺ” N. Bobbio clarified this distinction in many of his works.

[^134]: Aristotle expressly states that movement is between things of the same genre, and not between things of different genre (see also *Phys.* I, 5). Nevertheless, because the contraries are particular contradictories (as it will be demonstrated), only in a particular sense this thesis is true: in the passage of *Metaphysics* 1011b30-35 Aristotle expressly states that becoming is between contradictories (from good to non-good and vice versa). G. Reale translates the term ἀντικειμένα with “contrari”, but “contrario” is ἐναντιόν. Ἀντικειμένα is used by Aristotle to talk of opposition in a generic meaning (i.e. in the *Categories*). See also *De int.* 23b14, where Aristotle writes that movement is between opposites, and not contraries.
cause maybe it's necessary for the same thing to suppose that the good isn't good”\textsuperscript{135}

Here's already evident the inclusion of contrariety in contradictoriety: the contrary is a special contradictory, so as good is a special “not-bad” (to be precise, the fairest “not-bad” that there is)\textsuperscript{136}.

The second passage can be found in the book Γ of Metaphysics: “And, because it's impossible for the contradictories, referred to the same thing, to be true together, it's evident that the contraries can't also be together in the same subject. In fact, one of the contraries is not only a contrary, but also a privation. Now, privation is the negation of a specific genre of properties of the substance”\textsuperscript{137}.

That “one of the contraries is also privation” means, in the elastic and not always strict language of Aristotle, that of two contraries one always expresses the negation of the other (exactly, "the nega-

\textsuperscript{135}"Ἡ μὲν οὖν ὅτι οὐκ ἐγαθόν τὸ ἐγαθόν τοῦ καθ' αὑτό ὑπάρχοντας ψευδής, ἢ δὲ τοῦ ὅτι κακόν τοῦ κατά συμβεβηκός, ὡστε μᾶλλον ἂν εἴη ψευδής τοῦ ἐγαθοῦ ἢ τῆς ἀποφάσεως ἢ ἡ τοῦ ἐναντίου [...] ἢ δὲ τοῦ ὅτι κακόν τὸ ἐγαθόν συμπεπλεγμένη ἔστιν. Καὶ γὰρ ὅτι οὐκ ἐγαθόν ἁνέργη ἢ σον ὑπολαμβάνειν τὸν αὐτόν”.

\textsuperscript{136}See also, on the topic, E. Severino, cit. p. 26-27, and particularly p. 26: “solo perché si pensa che il bene è non bene si può pensare che sia quel certo non bene che è il male”. Severino has remarked a particular meaning of contradic-

\textsuperscript{137}Met. 1011b15-20:"ἐπεὶ δ’ ἄδυνται τὴν ἀντίφασιν ἀμα ἀληθεύεται κατὰ τοῦ αὐτοῦ, φανερῶς ὅτι οὐδὲ τάναττα ἀμα ὑπάρχειν ἐνδέχεται τὸ αὐτῷ τῶν μὲν γὰρ ἐναντίων θάτερον στέρησις ἢστιν ὁχὴ ἢτιν, ὁσίας δὲ στέρησις. Ἡ δὲ στέρησις ἀπόρωσις ἢστιν ἀπὸ τοῦ ἀρματών γένους”. This passage is full of meaning and will be used also as a tip for a critique of Lukasiewicz.
tion of a precise genre of property of the substance”). This is another way to say that the contraries are special contradictories, considering that they always are re-connectable to the fundamental contradictory opposition, of which they are extremes.\textsuperscript{138}

b) The last thing left to do, is to establish the nature of the relationship between these two formulations of the PNC, and to show why the second one, just analyzed, must be considered “weaker” than the first one. The PNC, in its strict formulation, states that the same predicate cannot belong and together not belong to the same subject under the same aspect. The weak formulation of PNC, instead, states that it's impossible for the contraries\textsuperscript{139} to belong together and under the same aspect to the same subject. These are the fundamental differences:

1) The strict formulation regards the belonging and the not-belonging; the weak formulation regards different belongings. Using an already cited Aristotelian passage, we can say that the first formulation is about contradiction itself, the second is about contradictions for accident. With that is established, ultimately, that the PNC in its strictest formulation doesn't refer to two different predicates (contraries, contradictories, relatives or else), but to the same predicate, and to its inherence and not inherence to a substrate.

2) Going further with the first point, the strong formulation of the PNC can be reconnected immediately and perfectly to the propositional one. That the same predicate can't belong and together not belong to the same subject means that two contradictory pro-

\textsuperscript{138} The translation of the passage by I. Düring (cit. p. 686) is not correct from this standpoint, because it doesn't keep an eye on the precision that Aristotle keeps in distinguishing the various kinds of opposition between terms. Also G. Pasquale (cit. p. 18) doesn't understand the meaning of this passage, because he agrees with the synonymous usage of these two terms by Aristotle, where he wanted to distinguish them and to show their relationship.

\textsuperscript{139} It has been specified, in the beginning of the paragraph, that there are various weak formulations of the PNC. It's possible to substitute “contraries” with “contradictories” or “opposites”.

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positions can't be both true. The same is not for the weak formulation: translated in a propositional field it would regard contrary propositions, but not contradictory, even if the contrariety of the propositions implies their contradictoriety (see previous note).  

3) The strict formulation of the PNC means exactly what is meant by the law of the excluded middle. It's not the same for the weak formulation, and that's easy to show. It's certainly true that two contrary predicates cannot belong together to the same subject; but from this doesn't follow that of the two one must necessarily belong to the subject itself. The same man can't be together good and evil, but he must not necessarily be good or evil; a building can't be together white and black, but not necessarily must it be either white or black.  

4) Substantially, the weak formulation of the PNC is a particular case of the strong formulation. They're not equivalent formulations, and they don't have the same meaning, nor the same validity. The fact, that often Aristotle uses the weak formulation of the PNC in order to defend it, doesn't mean that he identified them, but simply that in the particular context, in which he was, that formulation was sufficient for was he needed to demonstrate.

3. In 1910 the famous logician Jan Łukasiewicz, well known for his historic and not only technical knowledge of logics, wrote an essay on the “principle of contradiction” in Aristotle, meaning with this expression what here is called the PNC. The text is inspired by the agreeable principle that “it's a bad thing that in philosophy there are untouchable principles”, and for this reason it's about one of the great principles of the history of western thought,

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On this regard see also the chapter fourteen of De Interpretatione.

\[141\] J. Łukasiewicz, De principio di contraddizione in Aristotele, Quodlibet, Macerata 2003. Let's remark that in the past this principle was called the “principle of contradiction”, that is the principle on which one falls in contradiction. Now the more accurate denomination of “principle of non contradiction” is preferred. On this topic see also as a specimen G.W.F. Hegel, Scienza della logica, trad. Di A. Moni, Bari, Laterza, 1968, 2° ed., p. 463; A. Rosmini, Logica, a cura di E. Troilo, Milano, Fratelli Bocca, 1942, XX, Vol. I, p. 137.

\[142\] Ivi, p. 15

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that, in the author's opinion, were never taken into consideration with an original spirit of critique, but only with dogmatic devotion. The nature of this essay is double: on one side it's an historical work, in which Lukasiewicz analyses the Aristotelian exposition of the principle and the way in which Aristotle defends it; on the other side it's a text that features strong theoretical and critical points. In these point, the author “tests” the PNC from a modern standpoint, and shows not only its weak aspects, but also the “living” and still useful aspects of it.

After the analysis of the last chapters, thought, the conclusions of the present essay seem to contrast with the ones of the Polish logician. A comparison between the arguments of both essays can be useful to underline some special features of the PNC, which haven't been considered yet, or at least to confirm the validity of the arguments exposed in the precedent pages with the support of a fundamental (even if old) text on the theme.

The main criticism that Lukasiewicz expresses against the principle is that it at the same time needs to be demonstrated, and is not able to be demonstrated\textsuperscript{143}. This aspect, with others, will be considered in the following chapters; for now it's important to analyze the statements of the Polish logician on the Aristotelian formulation of the PNC. They can be summarized in three points:

1) There's not only one PNC, nor different formulations of the same principle, but three different principles: the ontological, logical, and psychological principle of non contradiction\textsuperscript{144}. The first one is the following: “No object can own and not own the same attribute at the same time”. The logical principle is “It's impossible

\textsuperscript{143} Ivi, p.54: “Il principio di contraddizione è un giudizio generale ed esprime la relazione tra un dato oggetto e la proprietà di non poter possedere e non possedere contemporaneamente lo stesso attributo. La verità di questa relazione non sta in se stessa, ma esige incondizionatamente una prova. Perfino quei filosofi che ritengono ovvio il principio di contraddizione, fondano la sua verità non su di esso, bensi sull’evidenza, ossia su un fatto psichico che percepiscono di fronte a tale principio. Il principio di contraddizione dunque non è ultimo, e chiunque lo accetti oppure voglia convincere altri ad ammetterlo, deve dimostrarlo.”

\textsuperscript{144} Ivi, p. 24: “…le tre formulazioni non rappresentano un unico principio, espresso con delle parole diverse, ma sono tre principi diversi”.

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to be both true for two statements, of which the first gives to the object the attribute, that the second negates to it”. The psychological principle is: “Two convictions, to which contradictory statements correspond, cannot be together in the same mind”.  

2) The PNC is not the “ultimate principle”, because it's founded on the principle of identity, and the principle of identity, again, is founded on the definition of true judgement.

3) The PNC and the so-called “principle of double negation” are two different principles, they can't be identified so as the three different principles of non-contradiction.

Now we can consider separately the three arguments, and view the impact and the validity that they have in the Aristotelian exposition of the PNC.

3.1. “Aristotle formulates the principle of contradiction in an ontological, logical and psychological meaning, even if he never expressly states these distinctions. For the moment we won't take into consideration the psychological principle: in the following chapter it will be seen how the introduction of it is caused, in Lukasiewicz's essay, by a substantial misunderstanding of the Aristotelian text.

The other two principles are left to the examination and in order to consider them, we must introduced the definition that Lukasiewicz gives of synonymity. Are synonymous two judgements that express with different words the same thought? The two judgements “P has c” and “Q has d” are synonymous if Q and P mean the

146 Ivi, p.45:“Tra i giudizi generali esiste un principio che, ancor più del principio di contraddizione, possiamo considerare ultimo: è il principio di identità”.
E p. 51: “Il principio di identità non è definitivo, perché lo si può dimostrare in base a un altro principio, che ne è la ragione: tale principio è la definizione di giudizio vero”.
147 Ivi, p.46:“Possiamo provare che nessuno di questi tre principi: di identità, di contraddizione e di doppia negazione, esprime lo stesso pensiero e di conseguenza sono tutti eteronimici”.
148 Ivi, p.15.
same thing, and c and d mean the same property.\textsuperscript{149} Two synonymous judgments are always equivalent (they imply each other), but two equivalent judgements are not also always synonymous. The logical and the ontological principle of contradiction are, for Lukasiewicz, not synonymous: “In the ontological principle of contradiction it's talked about objects, in the logical one of judgements”\textsuperscript{150}. They are, nevertheless, equivalent, and Lukasiewicz demonstrates that using the passage 18a39-18b2 of \textit{De interpretazione}: “In truth, if it's true to say that an object is white or that it's not white, it will necessarily be white or not white...if [an object] is white or isn't white, it was true to affirm or to negate it”. A few words later he specifies, though, that “Aristotle would have called this a logical and not a real equivalence”, quoting the passage of \textit{Metaphysics} Θ 10, 1051b69: “Not because we think you white, you're really white, but because you are white we, that affirm that, say the truth”. The conclusion of the Polish logician is the following: “Being is a logical reason of the truth of judgements, and a real cause of their enunciation; the truth of judgements is, instead, only a logical reason and not a real cause of being”\textsuperscript{151}

Now we have all the instruments to consider this theory. First of all, it's necessary to make precise that the notion of synonymity used by Lukasiewicz is not Aristotelian: for Aristotle the synonymity was between terms, not between propositions. Moreover, it has to be specified that saying that “the logical principle refers to judgements, the ontological one to terms”, is not fully correct: it's naturally true, but we've already seen that the PNC is not about objects, but their own or not specific determinations. In order to be correct, so, the “logical” PNC treats of the truth of judgements; the “ontological” PNC treats of the belonging of the attributes to the objects. So, it has already been stated how between these mo-

\textsuperscript{149} \textit{Ivi}, p.23. Further in the page there seems to be a definition of synonymity between judgements, but probably the text of the polish logician is poorly translated, or Lukasiewicz made a mistake. The definition that he gives is, in fact, the definition of equivalence, and not of the concept of synonymity. The structure of the text, anyway, makes an editing error probable.

\textsuperscript{150} \textit{Ivi}, p.24.

\textsuperscript{151} \textit{Ivi}, p. 26
ments there's a greater link than the one that Lukasiewicz shows: the truth of a judgements means the existence of a fact, that is the belonging of an attribute to a subject. Saying of a proposition that it's true, or false, is not a question of logics, but of ontology (or its region, that's interested to the kind of being that is subject of the proposition)\textsuperscript{152}. That these two statements are not synonymous, in the meaning that Lukasiewicz gives to the term, doesn't imply that they can't say the same thing: that's in a really similar way to the one that allows to heteromorph expressions to have the same meaning, or un-identical logical proposition to express the same content (according to the principle of extentionality).

An example is particularly useful: let's take the two statements “the pen is on the table” and “the statement “the pen is on the table” is true”. The first statement gives an attribute to a subject, the second one gives a truth value to a proposition: superficially these two propositions seem to refer to different domains, and consequently to pertain to different fields. The first one is an “ontological” proposition. But can it really be said, that the first is a “logical” proposition? It's evident that these two propositions want to express precisely the same content, and it's also evident that their truth depends from the same fact\textsuperscript{153}.

\textsuperscript{152} Banally, an affirmation that didn't state that the proposition “water boils at 30 degrees” is always false would not be a proposition on logics, but on physics.

\textsuperscript{153} To clarify all this it one can use the distinction, made by I. M. Bochenski, between “logical” and “metalogical” formulation of the PNC: “A whole book of Metaphysics (Γ) is devoted to the principle of contradiction […] and there are numerous formulations of it in his other works. Those formulations may be divided into logical and metalogical” (I. M. Bochenski, Ancient formal logic, North-Holland publishing company, 1951). Let's remark that also Bochensky talks about different formulations of the same principle. This because, exactly, changing the form of the exposition, what lasts is the fundamental meaning of the principle. It's possible to create infinite metalogical formulations of the PNC: this, nevertheless, wouldn't change its unity. The religious polish man, anyway, seems to agree with Lukasiewicz when he writes (La logica formale, Ein-audi, Torino 1971, p. 86) that Aristotle was conscious of the differences between the two formulations of the principle of non contradiction. Whether he agreed or not, this doesn't change that he has exactly written: Aristotle well knew that words are not things. But the distinction doesn't affect the absolute unity of the two formulations.
It has to be remarked, though, that there's a difference between this example and the logical PNC: in the first case it's a simple affirmation, in the second case it's a proper law, that rules a determined aspect of the connection between affirmations and of the attribution to them of truth values. Our example can be reconnected to the domain of reality, but in the case of the logical PNC we have a formal, logical affirmation, that has nothing to do with ontology\textsuperscript{154}. This is precisely the opinion of Lukasiewicz: Aristotle has formulated a law of being and a law of thought, and admitted that between them there's a reciprocal implication\textsuperscript{155}. For the Polish logician it's possible to demonstrate that from the logical principle derives the ontological one and vice versa, even if he remarks that Aristotle gave more importance to the ontological one. It's obvious that this opinion is possible only if one thinks to deal with two different principles. And it's confutable: the “logical” principle can't lead to the ontological one as Lukasiewicz says. For him, “that two contradictory attributes belong to the same object” is impossible, because that would imply that two contradictory statements both be false, and this is exactly impossible\textsuperscript{156}. But it's Aristotle itself (in a passage quoted by Lukasiewicz too) that specifies that it's not reality, that is founded on Logos, but the Logos, that is founded on reality: it's impossible for two contradictory judgements to be both true because world is not contradictory (and these two affirmations say the same thing), but it's not impossible for the world to be contradictory because there's a law of the Logos that prevents that\textsuperscript{157}. As the PNC is split in two, and from one of the two principles is taken away its ontological value, the equivalence is no longer possible. The relation of “logical reason” that

\textsuperscript{154} In summary, Lukasiewicz treats the logical principle as something “postulated”, an axiom in the meaning of the modern formal logic, something that, in actuality, has nothing to do with.

\textsuperscript{155} Ivi, pp. 24-25.

\textsuperscript{156} Ivi, p. 25.

\textsuperscript{157} Maybe it could be said that the Philosopher wouldn't have hesitated to build a contradictory logic if he saw contradiction in reality. Moreover, he never interprets the PNC as Lukasiewicz does, as if it was one of the fundamental sentences of an axiomatic system. It's enough to note that the only axiomatic system built by Aristotle, his syllogism, doesn't have the PNC as an axiom.
the Polish logician establishes between being and truth is unnecessary and vicious\textsuperscript{158}.

That Aristotle didn't separate the two principles, and that he believed in their substantial unity, is witnessed by various passages of its (“his”?) works. We've already seen, in the first chapter, the context of many uses of the PNC: well, it's singular, if we accept the argument of Lukasiewicz, that Aristotle often uses, in the Organon, the ontological formulation of the PNC to solve logical and propositional problems, or to clarify some passages of its syllogistic. But the passage in which the identity between the two principles is evident is in the Metaphysics, and has been already quoted: “and because it's impossible for the contradictories, referred to the same thing, to be true together, it's evident that neither the contraries can be together in the same object. In fact, one of the two contraries, more than contrary, is also privation”.\textsuperscript{159}

The main error in the interpretation of this passage is to believe that it pertains to the implication between logical and ontological aspect. But here Aristotle wants to solve the question of the relation between contraries and contradictories, and to do that uses indifferently the “ontological” and the “logical” formulation of the PNC, as if it was exactly the same principle.

Concluding: it has been already specified that Aristotle knew the difference between logic and reality. The formal aspect of its logic consists nevertheless in his syllogistic, that is different from the treatment of PNC. The PNC is a law of reality, a principle that concerns the whole being: that implies that it concerns also the correct way of reasoning, because the correct way of reasoning mirrors the being, and the thought, because thought is a part of being.

This part of Aristotle's logic (or, we could simply say, the Aristotelian logic) is a logic that is modeled upon reality, or at least

\textsuperscript{158} Vicious exactly because according to Aristotle a “logical PNC” couldn't be something assumed independently from the observation of reality, and would so become something deduced from the ontological PNC, and that wanted to deduce it itself.

\textsuperscript{159} Met. 1011b15-19.
aims to be modeled upon it. If we take the so-called “propositional formulation” of the PNC as a law of language or of thought standing by itself, we have to accept that it's a law based on the ontological formulation of the PNC. If instead, as in this essay, we take the two formulations (propositional and functional) as two ways to express the same principle, then it's possible to understand how two different expositions of a law of the real that, consequently, is also a law of thought and of correct argumentation.

3.2. The second point on which the Polish logician argues with the Philosopher is in chapters VII and VIII of his essay on the principle of contradiction: according to Lukasiewicz, as already briefly said, the PNC is not the ultimate principle, because it requests another principle before it, without which it would not have meaning: it's the principle of identity (from now POI). Secondly, in the following chapter Lukasiewicz adds that the POI itself is not ultimate, but is founded on the definition of true judgement, that is the authentic “unconditioned” in the Aristotelian logic structure.

Lukasiewicz's argumentation involves also the so-called “principle of double negation”: but of it will be spoken in the next paragraph. Here we're going to analyse just the first two arguments, referring to what can be found in Aristotle's works.

a) Aristotle never formulates the POI, at least not expressly: nevertheless, various passages of his works can be taken as sufficient formulations of it, so that they can be used to verify Lukasiewicz's thesis. For example, the statement that “it's neces-

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160 I simply mean that aristotelian logic is based on a certain interpretation of reality.
161 For reference see also note 88.
162 See also Lukasiewicz, cit., p. 45, e I. M. Bochenski, cit. p. 38: “…we find no principle of identity in the preserved writings of Aristotle”. 
sary for being to be when it is, and not to be when it is not"\textsuperscript{163}. The critiques of the Polish logician are as follows:

1) The POI is presented, usually, as the “positive side” of the PNC. But it's impossible for a negative judgement to be synonymous of a positive judgement\textsuperscript{164}. In this case it's useful to repeat what already stated for the formulation of the PNC: that two formulation are heteronymous (in the meaning in which Lukasiewicz uses this term) doesn't exclude that they can have the same meaning, that is that they say the same thing. It has to be considered, moreover, that Lukasiewicz uses pure logical arguments, but the logic structure of Aristotle's speech is not formalized: this is important to note in order to differentiate between the “historical” passages of Lukasiewicz, and his theoretical critiques\textsuperscript{165}.

It's possible, anyway, to repeat the critique already expressed against Lukasiewicz's concept of synonymity: basing on it, stating “2+2=4” is different from stating “it's impossible for the result of the operation '2+2' not to be 4”.

\textsuperscript{163} De int. 19a23-24: “Τὸ μὲν οὖν εἶναι τὸ ὄν ὅταν η, καὶ τὸ μὴ ὄν μὴ εἶναι ὅταν μὴ ἦ, νάγκη”. On this affirmation as a formulation of PNC see also Severino, cit., p. 67: “Tenendo presente che 1) questa affermazione si riferisce all’ente in quanto ente (e cioè si riferisce anche agli enti eterni, per i quali la necessità di essere “quando” sono non è limitata ad un certo tratto finito del tempo) ; che 2) la necessità è propria dei principi o di ciò che da essi discende ; che 3) tale affermazione è enunciata come qualcosa che non ha bisogno di dimostrazione, e quindi la sua è la necessità dei principi e non delle loro conseguenze; e che 4) nel linguaggio di Aristotle l’essere dell’ente è tanto l’essere esistenziale quanto l’essere copulativo con predicato sottinteso: tenendo presente questo insieme di considerazioni è consentito affermare che il passo ora riportato è una formulazione esplicita di ciò che è stato chiamato principio d’identità”. Earlier (p. 33) Severino reconnected to the POI what Lukasiewicz also called “definition of true judgement”. In order to avoid misunderstandings, this won’t be used.

\textsuperscript{164} Lukasiewicz, cit. p. 46.

\textsuperscript{165} For a complete demonstration of the semantic unity of POI, PNC and law of the excluded middle see M. Malatesta, Sulla formulazione unica dei principi di identità, non contraddizione e terzo escluso nella logica proposizionale, I, Pizzi, Reggio Calabria 1979; II, Tipografia artigianale, Latina 1979; III, Giannini, Napoli 1981
2) Lukasiewicz expresses the POI and the PNC as hypothetical principles. They are:

POI: If P has c, then P has c.

PNC: If P is an object, then P can't have c and together not have c.

The critique of the Polish logician is the following: in the PNC we find the term “object”, but this doesn't happen in the POI. So, the PNC can't express the same meaning of the POI. This is the answer to the critique: at the beginning of his essay, Lukasiewicz states to refer, with the term “object”, to anything that is not nothing. 166 “Object”, in the end, means “being”. So, it has been seen in the first chapter how Aristotelian logic is a logic of what is, and not of what isn't. It's necessary, consequently, also for the POI to be referred to beings, and not to not-beings; it's necessary for the term “object” to be included in its definition: “If P is an object and if P has c, then P has c”. 167

Another critique of Lukasiewicz on the unity of POI and PNC is the following: the second uses two notions, logical product and negation, that are not useful for the formulation of the POI. 168 But we have to remember the way in which Aristotle “formulates” the POI: it's necessary for what is to be, and for what is not to not be. Well, the notion of necessity implies the notion of negation 169 (ne-

166 Lukasiewicz, cit. p. 19: “Intendo per oggetto qualsiasi cosa che sia ‘qualcosa’ e non ‘un niente’”. There is a certain ambiguity in the usage of the term “something” by Lukasiewicz. Probably he uses it as synonymous of “something significant”. From this standpoint a unicorn would be “something”, that is an object.

167 More than this, the banal remark that in order to have an attribute P must be an object. This is said by Lukasiewicz himself (cit. p. 21: “intendo per attributo tutto ciò che si può dire di un oggetto”), falling evidently into contradiction.

168 Lukasiewicz, cit. p.48

169 A fundamental problem is determining the concept of negation can be seen in these pages. At page 64 of his essay, Lukasiewicz seems to refer to a meaning of negation, according to which it doesn't “remove”, or doesn't exclude, affirmation. The statement of the negation wouldn't remove the statement of affirmation. But in my opinion is this sense of negation is meaningless, and thus the
cessary = impossible that not = not possible that not\(^{170}\), confuting consequently the first part of this second critique. Regarding the second part, stating the necessity of identity means stating that it's impossible for being not to be identical with itself\(^{171}\). Now, in order to express the non-identity of a being with itself it's necessary to use the logical product. In fact, if it's necessary for P, that has c, to have c, then it's impossible for P, that has c, to not have c, which is to say that it's impossible for P to have and not to have c. In this case the use of logical product is necessary, because it's only when and if P has c that it's necessary for it to have c. If, P not having c, didn't P have c anymore, the POI wouldn't be negated. It's evident, consequently, that if on the surface the two principles say different things, in their concrete form, that is in their semantic wholeness (totality of meanings expressed and implied) they say exactly the same thing. That could be expressed, saying that the PNC is not really the PNC if it doesn't contain semantically (means) the POI, and vice versa.

Summarizing: affirming that, if P has c, then necessarily P has c, means to affirm that it's impossible for P to have and not to have c; on the other hand (also considering the already stated unity of

corresponding meaning of affirmation; but this doesn't pertain to this work. In this note I simply want to specify that the term “negation” is meant in its genuine sense, according to which the statement of the negation removes the statement of the corresponding affirmation. On this topic see also G. Priest, Che c’è di male nelle contraddizioni?, in AA. VV., Scenari dell’impossibile, pp. 40-41: “Asserire una negazione non è necessariamente formulare un diniego (=asserzione della negazione) … si può formulare il diniego di qualcosa senza assereire una negazione”. For the american logician the critique is exactly the same, and it's not true that this problem is “a problem of the classical logician as of the dialectic logician (=negator of PNC)”. This last must explain what this new meaning of negation means, because he introduces it.

\(^{170}\) See also De int., ch. 13.

\(^{171}\) Whatever is the sense used to interpret negation, what is sure is this: the necessity of identity of being with itself means that it's something that can't be removed. Now, whatever the meaning of negation is, it will be necessary to deal with this removing – and it will be exactly this removing to remain untouchable beyond every determination of the concept of negation (and we have already said that this explanation is not something the researcher of the PNC must do).
PNC and law of excluded middle) stating that P can't have and together not to have c means that P, as long as it has c, must have c, that is to say that can't not have c.  

\[ \Phi_x \rightarrow \Phi_x = N(\Phi_x \land N\Phi_x) \]

From all this derives that POI and PNC are two unitary principles, so that they have the same meaning: the PNC can't be expressed without expressing the POI, and vice versa.

Closing this first part on identity, some consideration. First of all, it's evident that the unity of POI and PNC can be stated only in a context that makes a “classical” use of negation. What Lukasiewicz states many times in his essay, that is that the affirmation “if P has c, P necessarily has c” doesn't imply “it's impossible for P to have and not to have c”, can be valid only if the opposition between affirmation and negation is rejected.

\[ \text{This equivalence between identity and double negation is possible on the basis of the rejection of the } \text{Lukasiewicz's concept of synonymity. We must specify that the use made by the Polish logician of negation is useful in a situation in which is not accepted the classical (aristotelian) use of negation} \]

I explain again the usage of symbols: “Φ” is a variable that can mean any feature, that the object x owns. “Φx” consequently means “the feature Φ belongs to the subject x”; “\(\rightarrow\)” is the implication symbol. All the other symbols keep their meanings.

On this topic a specification about the position of Severino on the link between the two principles is really useful: Severino (cit. p. 68) states the equivalence but the non-identity of POI and PNC. This is agreeable in this essay, but only as long that means that the two principle don't expressly say the same thing. But Severino himself confirms what was said, when he states the necessity of their union, and writes that “il principio più saldo non è il p.d.n.c., separato dal principio d’identità, e nemmeno viceversa”. This exactly means that stating the PNC, as separated from the POI, is in its abstract form semantically incomplete and not absolutely valid. Ultimately: superficially the PNC and the POI obviously state different things. But the semantic content of PNC in its concrete form is the same of the POI, in its concrete form, and this because the PNC, concretely considered, states also the POI, and vice versa (see also M. Malatesta, cit., parte III pp. 35 e 40). This also is perfectly compatible with aristotelian thought.

Anyway, because the opposition between affirmation and negation is also their incompatibility, and because the PNC is the authentic meaning of this in-
has to be established, and it will be done in the analysis of the Aristotelian defense of the PNC, is if the rejection of the opposition between affirmation and negation can resist to the principle. But more than this, it has to be specified that the critiques of Lukasiewicz don't consider the ground of the Aristotelian exposition, and come from a theoretical atmosphere that is completely different from the one of Aristotle. This means that Lukasiewicz writes about a POI that is not Aristotle's POI.

Moreover, the thesis of E. Berti is valid, according to which it's not correct to say that the POI is a prerequisite of PNC, because the last one requires only the notion of “identical” (αὐτό), that has nothing to do with the POI.176

b) From all this it's possible to deduct that the POI can't be a prerequisite of the PNC, because 1) it's only a different formulation of it 2) this last statement requires the notion of identity, but not the POI. Now we have to consider the second part of Lukasiewicz's thesis, according to which the POI is based on the definition of true judgement: that means, on the basis of what already said, that POI and the PNC are based on it.

Two preparatory considerations: the first is how is the identity “required” from the PNC just a notion, and not a principle. The same is for the truth of judgements. From this standpoint, the PNC is not compatibility, negating the opposition between affirmation and negation means to negate the PNC (and for this reason is said that in the aristotelian defence of the PNC the validity of this theoretical postulate is proved). What is important is that, preliminarily negating this opposition, it's already a POI that is abstracted by the PNC, separated from it, and consequently a POI and a PND that are not the ones of Aristotle. Moreover, in order to avoid misunderstandings, it's evident that the usage of this opposition in the explanation of the meaning of the PNC can be considered a *petitio principii* only as long as the expositions of these two principles are separated and not simultaneous moments.

176 E. Berti, cit. p. 110: “Non si può nemmeno dire che il p.d.n.c. aristotelico supponga il principio d’identità, almeno quello formulato nella maniera tradizionale, sia perché Aristotele stesso lo considera come abbiamo visto, il primo fra tutti gli assiomi, e dunque tale da non supporne altriprima di sé, sia perché ciò che esso suppone è la nozione di identità, così come suppone altre nozioni (impossibile, appartenere, tempo, aspetto, ecc.), cioè delle nozioni primitive, non dei principi, ossia non una proposizione, una regola, una legge.”
based on the notion of “true” more than on the notion of “together”, “impossible”, “belong”\(^{177}\). The second consideration is that if the definition of true judgement was something anterior to the PNC and to the POI (and thus not subjected to them), it couldn't be the definition of true judgement. Because it aims to be itself and to exclude its not-being-itself, the definition of true judgement is already subjected to the PNC (that is “transcendental”\(^{178}\).

Lastly, a consideration on the way in which Lukasiewicz tries to demonstrate the POI on the basis of the true judgement. I quote the passage in order to discuss it: “the judgement that assigns to an object the attribute that it owns is true; the judgement that negates to an object the attribute that it doesn't own is true. From these definition follows directly that if an object owns an attribute, so it’s true that it owns it; and if it doesn't own it, so it's true that it doesn't own it. These propositions are true, because I exactly in this way define truth”\(^{179}\).

Many conclusions can be taken: first of all, in order to say “If P has c, then it's true to say that P has c”, it's necessary to admit that “if P has c, P has c” and “if it's true that P has c, than it's true than P has c”. This because, if not, we would deal with a P that could not have c, and the truth of the whole proposition could be compromised. But more than that, this demonstration can be considered a formulation of the POI\(^{180}\): we already saw how the facts

\(^{177}\) See also Berti, cit. p. 110

\(^{178}\) See also Severino, cit. p. 69-70: “Basti richiamare che se il principio di identità [e quindi il PNC] fosse cosí fondato, la ‘definizione di giudizio vero’, precedendo concettualmente il principio d’identità, non potrebbe avvalersene, cioè non potrebbe essere una definizione che è necessario che sia una definizione di giudizio vero; ossia non potrebbe escludere di non essere una siffatta definizione”.

\(^{179}\) Łukasiewicz, cit. p. 51.

\(^{180}\) It could also be said that the definition of true judgement is a formulation of the POI, as i.e. father Gabryl and Severino think. For this last, the definition of truth is a formulation of the POI exactly because true is thinking the identity between what is connected and its being-connected, and what is separated and its being-separared. Shortly, it’s not true of every P to state that it has c, but only of that P that has C: consequently true is not “P has C” but “P, that has C, has C”.

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are cause of the truth of affirmation, but not vice versa. According to that, the implication that Lukasiewicz makes between truth and being when he writes that “if it's true that it owns it, that it owns it” is right only if it's admitted that “P has c” and “it's true that P has c” express the same meaning. It's true that P has c doesn't express a logical or linguistic meaning, but an ontological one. The demonstration of Lukasiewicz would then become the following: “If an object owns an attribute, then it's true that it owns it, that is to say that it owns it; and if it doesn't own it, than it's true that it doesn't own it, that is to say that it doesn't own it”. It's evident that this is no longer a demonstration, but a simple affirmation of POI.

3.3. Another critical point of Lukasiewicz's essay is the relation between the PNC and the so-called “principle of double negation” (PDN). It's important to clarify this relation, because in the history of thought many philosophers have used the PDN as as synonym for the PNC. Moreover, Aristotle itself seems to use the two for-

181 This on the basis of the same principle previously stated, that “the pen is on the table” and “the sentence 'the pen is on the table' is true” express the same meaning.

182 All this operates on the basis of the given definition of true and false: if true is the union of what is not united, stating the truth of union is stating the union, and stating the truth of judgements means stating the existence of the corresponding fact. To abstract these two moments, as if they were two connected but different phases, means losing the sense of truth (see also Tommaso d’Aquino, Sulla verità, Bompiani, 2005, p. 116: “Augustinus in libro Soliloquiorum dicit quod verum est id quod est; sed id quod est nihil est nisi ens; ergo verum significat omnino idem quod ens”).

183 It's good to specify the thesis of Lukasiewicz: he doesn't exactly state that the POI is based on the definition of true judgement, but that it's possible to demonstrate it on the basis of this definition (that is what has been confuted). Was it not so, and did he not mean that, it could be simply answered to him that the formulation of the POI doesn't make use of the notion of truth.

184 For example Kant (Critica della ragion pura, p. ), Gentile (Sistemi di logica come teoria del conoscere, Le lettere, 2003, p. 181 ), Severino (Fondamento della contraddizione, cit. p. 258 n.1; Essenza del nichilismo, Adelphi, Milano 2005, p. 44 ; La struttura originaria, Adelphi, Milano 2007, cap. 1)
mulations as synonyms\textsuperscript{185}, particularly in the defense of the principle. Lukasiewicz's critiques are in the chapters VII and X of his essay, and can be summarized in the following points:

1) The PDN is not a negative formulation of the POI, because, as Lukasiewicz thinks to have demonstrated at the beginning of his work, a negative judgement can never be synonymous of a positive judgement\textsuperscript{186}.

2) The PDN uses the concept of “negation”, which the POI doesn't need: consequently, they are not synonymous principles\textsuperscript{187}.

3) As for the relation between POI and PNC, the PDN doesn't use the concept of “object” nor the one of “logical product”\textsuperscript{188}.

4) The first elenctical proof of Aristotle defends the PDN, but not the PNC; because they're not synonymous. In fact, it's possible to find a case in which the PDN is valid and the PNC is not: it's the case of every contradictory object\textsuperscript{189}.

Here are the answers:

\textsuperscript{185} For example during the exposition of ελεγχος. see also Met. 1007a23-24: “εἰ γὰρ εσται τι οπερ ανθρωπω εἶναι, τοῦτο οὐκ εσταί μὴ ανθρωπω εἶναι η η μὴ εἶναι ανθρωπω”. See also Severino, cit. p. 268.

\textsuperscript{186} Lukasiewicz, cit. p. 46: “Nel capitolo II ho già menzionato che nessun giudizio affermativo è sinonimo del giudizio negativo; l'affermativo potrebbe essere al massimo equivalente di quello negativo, ma l'equivalenza e la sinonimia sono concetti diversi. Non è dunque ammissibile l'idea secondo la quale le formule: ‘a è a’ e ‘a non è non-a’ siano sinonimiche ed esprimano lo stesso principio”, and p. 23: “Nessun giudizio negativo è sinonimo del giudizio affermativo, perché l'affermazione significa una cosa diversa dalla negazione; una è altrettanto semplice quanto l'altra, e nessuna si lascia ricondurre all'altra”.

\textsuperscript{187} Ivi p. 48: “Possiamo inoltre esprimere il principio di identità senza ricorrere al concetto di negazione, mentre invece non possiamo formulare il principio di contraddizione [e il PDN] senza questo concetto.”

\textsuperscript{188} Ibidem: “senza ausilio di due giudizi costituenti un prodotto logico, si possono formulare sia il principio di identità che il principio di doppia negazione”.

\textsuperscript{189} Ivi, p. 63: “Aristote nella prima prova elenctica dimostrò al massimo il principio di doppia negazione, ma non dimostrò il principio di contraddizione [...] Esistono casi in cui il principio di doppia negazione è vero e il principio di contraddizione non è applicabile; diciamolo pure, è falso. Per trovare questi strani casi, dobbiamo esplorare gli oggetti contraddittori”.
The position of Lukasiewicz on synonymity has already been criticized. As already said, this strict separation between affirmation and negation derives from the implicit rejection, by the Polish logician, of the opposition between affirmation and negation, and of their incompatibility. But even if the link between these two elements is cut, even if a different meaning is given to negation, in order to make it compatible with the corresponding affirmation (so questioning the system of oppositions and implications usually possible between them), what is always true is that the affirmation has to exclude its own exclusion: what can't be put aside is that, independently from the meaning of negation given, the authentic meaning of negation as exclusion and removing cannot be avoided, and that from this negation the affirmation must be protected in order to remain itself. In a logical context, in which the negation doesn't exclude the affirmation, affirmation and double negation are not synonymous: but doesn't exclude that the affirmation is anyway the exclusion of its own exclusion, and consequently negation of that particular meaning of negation, according to which the affirmation is excluded.

Also here it's possible to recall the distinction between abstract and concrete POI. In the wholeness on its semantical content, the POI is also negation of non-identity; the necessity of being-itself for every being is its impossibility not to be itself, according to a meaning of negation as exclusion (and this meaning must be conserved, in order to make the POI, PNC and PDN concrete and not abstract principles\textsuperscript{190}). Saying that “A is A” means also to say that “A is not non-A”, as long as this means “A exists”.

\textsuperscript{190} The distinction between abstract and concrete can seem a heritage of the idealistic and neo-idealistic tradition (from Hegel to Severino) and particularly the use of this distinction in the exposition of the three principles is inspired by the works of E. Severino (see also on the topic La struttura originaria, Essenza del nichilismo, essays Ritornare a Parmenide and ΑΛΗΘΕΙΑ, and Tautotes). Nevertheless, this doesn't mean that Aristotle didn't mean the principles exactly in this way: he used negation in a classical meaning, as an exclusion. Consequently, a POI that doesn't exclude its own negation is not the POI of Aristotle, but a principle that doesn't say all that it should say, an abstract principle.
cludes the exclusion of its being-Α”. That then another sense of negation is introduced, it's not important\textsuperscript{191}.

3) That the PDN doesn't use the concept of logical product and of object is only apparently true. The elementary rule is that, if two elements are affirmed, the logical product of the two elements can be affirmed too. The PDN affirms that Π, that have c, can't not have c. But it affirms that only as long as Π has c: that is to say that it affirms the impossibility of the logical product of the owning and not owning of c for Π\textsuperscript{192}. On the question of the use of the term “object”, it can be repeated what was already said about the POI: Aristotelian logic is about existent things, and only them. Moreover, in order to have a predicate Π must be an existent object (and this is stated by Lukasiewicz himself)\textsuperscript{193}. It's consequently evident that the PDN makes implicit use of the premise “If Π is an object”, like every other logical principle. Its complete exposition is: “If Π is an object and if Π has c, Π can't not have c”.

4) The problem of existence of object is extended also to the last question: according to Lukasiewicz the case of contradictory objects is the only case in which the PDN is valid and the PNC is not. An example of the Polish logician is the K circle: it should at the same time have and not have a feature “b” (the possess of sides, that can be measured mit an algebraic number). I quote the passage in order to discuss it: “K has to have b, that means it must have sides, that can be described with an algebraic number. “It must

\textsuperscript{191} This on a historical and theoretical field. Historical, because Aristotle meant negation in a specific sense, and we have to keep the eye on it; theoretical, because beyond every determination of the sense of negation there's also that particular meaning of negation that is the exclusion of the affirmation.

\textsuperscript{192} It's true, however, that this argument is not expressly made by them, who use the PDN as a synonym of the PNC. It could be said that the PDN refers only to objects that own attributes, the PNC refers instead to objects of every genre, independently from their owning or not a certain attribute. The answer is simple: the formula of the PDN is “Φx → NNΦx”. Because it's possible to substitute every variable with another variable in all cases, the formula can be substituted with “NΦx→NΝΝΦx”, that means “if x doesn't have Φ, x can't not not have Φ, that is have Φ”.

\textsuperscript{193} See also supra, p. 19 n. 109.
have” means “it can't not have”. So, K can't not have b. Who uses the principle of double negation, must take for true these two judgements – Nevertheless, it's not true that K can't at the same time have and not have b: on the contrary, K has b and at the same time doesn't have b”. The possibilities are two: a) Lukasiewicz doesn't notice the mistake he falls in when he writes that in this case the PDN is valid. In fact, if it affirms that it's not possible that K doesn't have the predicate b, then the contradictory object K doesn't respect the PDN. b) Lukasiewicz uses two different meanings of negation, the first one being the normal sense of “exclusion”, the second being not specified, but necessarily present in order to be able to affirm that the object K can respect the PDN. It seems to be the second case: but if it's so, the PDN and the PNC are both valid, because the feature “not to have b” is made according to a meaning of negation that simply doesn't regard the two principles, because they refer (and this is admitted by Lukasiewicz himself) to a sense of negation that excludes the correspondent affirmation\(^\text{194}\); or both the principles are false, unless one of the uses the negation in a different meaning than the other (in that case they wouldn't be synonymous for obvious reasons, but one of them wouldn't either be the principle considered by Aristotle and this essay). Whatever the case, the premise is valid: Aristotle's logic is about beings. Lukasiewicz himself admits that his argument is meaningful only if K is recognized as an object, as to say something meaningful\(^\text{195}\). The possibility of such an attribution of meaning to “contradictory objects” pertains to the analysis of elenchos, that is left to a future work.

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\(^{194}\) This difference could be expressed as also stating that in a meaning negation is privation of affirmation, and in another meaning it's a second element that is added to it without referring to it.

\(^{195}\) This strongly agrees, for example, with the ontology of Severino, according to which the status of “being” is determined only by the criterion of significance.
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