A Short Note on Multiple Conclusion Logic in Physical Science

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Three score and ten I can remember well, within the volume of which time I have seen hours dreadful and things strange

Shakespeare, “Macbeth”

In a letter dated May 1952, Albert Einstein wrote to his friend Maurice Solovine, regarding what he called “la question épistémologique”: “Je vois la chose schématique-ment de la façon suivante:
(1) Les E (expériences immédiate) nous sont données. 

(2) A sont les axiomatiques d’où nous tirons des conclusions. Psychologiquement les A reposent sur les E. Mais il n’existe aucun chemin logique conduisant des E aux A, mais seulement une connexion intuitive (psychologique), qui est toujours: <jusqu’à nouvel ordre >.

(3) Des A sont déduites par voie logique des affirmations particulières S, qui peuvent pretendre à être exactes.

(4) Les S sont mises en rapport avec les E (vérification par l’expérience). Cette procédure, à y regarder de près, appartienne également à la sphère extralogique (intuitive) parce que le rapport entre les notions se présentant en S et les expériences immédiates E ne sont pas de nature logique.

Mais ce rapport entre les S et les E est (pragmatiquement) beaucoup moins incertain que le rapport entre les A et les E. (Par exemple, la notion chien et les expériences immédiates correspondantes). Si une telle correspondance ne pouvait pas être obtenue avec une grande sûreté (bien qu’elle ne soit pas logiquement saisissable), la machinerie logique serait sans aucune valeur pour la <compréhension de la réalité> (exemple, la théologie).”

In the present short note I will discuss only point (3), but would like to recall that points (1) and (2) and, partially, (4) were discussed in previous studies, in collaboration with the colleague and friend Ruggiero de Ritis: see, e. g., [1], [2], [3].

It is useful at this point to underline that in the present short note the notion of effective computability should be understood in the sense of the classic theory of recursive functions (see, e. g., [4]). As regards the notion of effectively computable scientific theory [in particular physical theory], the reference studies are by G. Kreisel, 1974 [5] and by Geroch e Hartle 1986 [6].

For a more subtle subdivision of effectively computable scientific theories see also [7], [8] and, in a different context, [9].

For examples of non recursivity in scientific theories see, e. g., [6], [10], [11], [12].
Coming back to Einstein’s point (3), it is useful to assert clearly that we are talking about what we could call the *logical structure* of the internal proceeding of a physical (or, more generally, scientific) theory T. Such *internal* logic should not be confused with the so called “*logic of scientific discovery*” supporting Popper’s falsificationism. The latter is concerned with the whole set of procedures of external control of a theory T at the test of experiment.

These concepts are completely *distinct* and yet *complementary*. Incidentally, I would like to recall that Einstein has been a precursor of Popper’s falsificationism. Already in 1922 [13] Einstein asserted: “The scientist who works theoretically is not to be envied, because nature or, more exactly stated, experiment is an inflexible and very little friendly judge of his work. It never says “yes” to a theory: at best it says “perhaps”, but in most cases simply says “no”. If an experiment agrees with the theory, for the latter this means ”perhaps”, and if it does nor agree, this means: ”no”.”

Once established a distinction between the external logic of scientific theories (i.e. Einstein-Popper’s logic of scientific discovery [13] [14]) and their internal logic, the latter is expected to be a deductive structure endowed with a recursive (effectively decidebole) set of axioms and hypotheses and with recursive rules of inference. Such rules of inference univocally and mechanically determine the consequence of a set of premises. Then, for each physical (more generally scientific) theory T, there is a logical theory τ (T) which expresses (from a logical point of view) T itself.

Let us do a step forwards. Obviously, given a physical theory T, a logical theory τ (T) capable of expressing T is such that each of its well formed formula (wff) α expresses a formula φα of T and the latter has an interpretation in the physical world.

Let us suppose now that a certain wff α belonging to τ (T) represents a non recursive φα.

What do we mean by this?
We mean that such \( \phi_\alpha \) (possibly representing a physical law) does not yield an effectively computable (recursive) procedure to obtain scientific previsions starting from data (initial conditions).

What to do? In a previous study, following a line of research initiated in [6] I wrote:” What do we mean by the expression “prediction from data” for an incomputable physical law? Let us consider a very general situation: a certain physical law suggests a procedure to go from data to prediction; this procedure, partially mechanical, implies some non mechanical steps. Of course, it is immediate to think of a Turing machine with oracle to model the previous situation. In fact this machine can carry on a computation, can ask for information to an external (non recursive !) source and, afterwords, go on mechanically.

We suggest that this information external source could be constituted by a set of “local” experimental results starting from which it is possible to construct an empirical formula: this empirical formula would provide the information necessary to bypass the non computable step of the procedure “from data to prediction” suggested by the law. We have in mind, e.g., Balmer’s formula of the hydrogen spectrum, the theoretical explanation of which has been given thirty years later (1913) by Bohr.

In our case the situation would be reversed: a physically sound and theoretically well founded approach would need the help of empirical formulas (and of approximate results) because of the non computability of some of his laws.” [15, p.264].”

At this point the situation is the following: a recursive derivation \( \delta \) allows to reach, in \( \tau (T) \), a wff \( \alpha \) starting from the set of axioms and hypotheses:

![Diagram](image-url)
The formula $\phi_\alpha$ of $T$, expressed by $\alpha$ in $\tau(T)$ and representing a physical law, is not recursive in that it does not provide a procedure which is effectively computable from data (initial conditions) to predictions. Then it is suggested, with a non-recursive empirical-experimental step $P$, to substitute wff $\alpha$ with wff $\beta$ which expresses in $\tau(T)$ a formula of $T$ representing a physical (empirical) law capable to provide a procedure effectively computable from data (initial conditions) to predictions.

But, then, one could have to face various problems. In my opinion the first problem is the following. As we have said, formula $\phi_\beta$ of the theory $T$ is empirically constructed recurring to adequate experimental instrumentation supported by theories which we consider, in a popperian language, to belong to our basic knowledge. Then, it is reasonable to think that in addition to $\phi_\beta$, we can obtain other empirical formulas capable to substitute $\phi_\alpha$. 
The preference given to one or the other will depend on various considerations: reliability of the theories on which the instrumentation is based, requested degree of approximation in previsions, maneagebility of mathematics used, ........

We would then face the following situation:

\[ \alpha \]
\[ \beta_1 \]
\[ \beta_n \]
\[ [A + H_{ip}] \]
\[ \delta \]

We would thus have more than one conclusion starting from premise \( \alpha \).

This leads to suggest, although with the necessary caution, that a multiple conclusions logic could perhaps be considered the most general internal logic of a scientific theory. But what do we mean by multiple conclusion logic? Here we will limit ourselves to quote Kneale who, referring to the famous work by Gentzen, 1934 [16], wrote: “if we wish to improve on Gentzen’s results, we must consider derivation or inference as a special case of something I shall call development. Unfortunately there is no recognized terminology for talking about this, but it can be described metaphorically as setting out the field within which the truth must lie if certain premises are to be accepted....... If a development has only one end formula, though this may be
repeated many times at the bottom of different branches, it is a derivation of the proposition expressed by that end formula. Just as a derivation is said to be valid if the premises involve the limits to which the development leads. Here I follow Carnap, who in his *Formalization of Logic* introduced the term “logical involution” for the relation that holds for two sets of propositions when it is impossible that all of the first should be true and all of the second false”. [17, p. 237].

Various Authors faced the problem of multiple conclusions in logic (see, e.g., Curry [18]) but the most general and complete work on this topic is by Shoesmith e Smiley [19]. These authors, in the beginning of their work wrote: “A multiple conclusion proofs can have a number of conclusion, say B₁….Bₙ: It is not to be confused with a conventional proofs whose conclusion is some one of the Bᵢ, nor it is a bundle of conventional proof having the various Bᵢ for their respective conclusions: none of the Bᵢ need to be “the” conclusion in the ordinary sense. This fact leads Kneale to speak of the “limits” of a “development” of the premises instead of the conclusions of a proof from them. We prefer to extend the sense of the existing terms, but hope to lessen one chance of misconception by speaking of a proof from A₁…….Aₘ to B₁…….Bₙ instead of a proof of B₁…….Bₙ from A₁…….Aₘ. The behaviour of multiple conclusions can best be understood by analogy with that of premises. Premises function collectively: a proof from A₁…….Aₘ is quite different from a bundle of proofs, one from A₁, another from A₂ and so on. Moreover they function together in a conjunctive way: to say that B follows from A₁…….Aₘ is to say that B must be true if A₁ and …and Aₘ are true. Multiple conclusions also function collectively, but they do so in a disjunctive way: to say that B₁…….Bₙ follows from A₁…….Aₘ is to say that B₁ or …..or Bₙ must be true if all the Aᵢ are true.”

Obviously, except in trivial cases or in cases irrelevant for the set of problems presented here, a multiple conclusion logic will turn out to be a non recursive inferential structure. Thus, the non effective decidebility of certain interpreted formulae will be eliminated only at the price of eliminating the recursivity of the logical structure. The way out from this problem, as we have said (see, e.g., [15]) is to recur to a sort of *oracle of Turing*.
constituted in our case by the physical experiment following the
way indicated in [6] and followed in [15].

In particular, as regards Quantum Mechanics, we remind
that:

Following the more shared point of view (Copenhagen
interpretation) Quantum Mechanics has a double aspect. The
temporal evolution of quantum states (ruled by Schroedinger
equation) is deterministic whereas the state function of a quantum
system gives the probability of obtaining a given value of a given
observable when a measure on this observable is carried on.

According to the Copenhagen interpretation it is the same
act of measurement (i. e. the interaction between the
microphysical quantistic system and the macrophysical measu-
rement instrument) which perturbs the system modifying its
evolution. This discontinuous change from an initial situation
where we have a superposition of various (possible, if you prefer)
states on a precisely defined final state (change due, as we said, to
the same act of measurement) is called “collapse or reduction of
Wavepacket”. Roughly speaking at this point (from a conceptual
point of view) there are at least two problems. First, the misterious
discontinuity of Wavepacket collapse which looks like an ad hoc
hypothesis. Second, the authentic weirdness of considering the
macroscopical object instrument of measure different in nature
from other macroscopical objects. In fact, the quantistic
microsystem, in this travel, interacts with various classical
macrosystems, but only the interaction with the classical
macrosystem instrument of measure would provoke the collapse.

Well, from the logical point of view discussed here the act
of measurement is precisely the element which determines the non
recursive step, empirical-experimental in character, which was
previously mentioned.

Then it is simple to see how the same argumentative
scheme can be applied to other interpretations of the formal
scheme of Quantum Mechanics, e. g. the interpretation called
“Many World interpretation [20].

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REFERENCES