Russell’s Description Theory:  
Unsurmountable Difficulties from a  
Rigorous Logical-Analytical Standpoint

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1. Russell’s Definite Description Theory

As is well known, the year 1905 was a milestone in the history of Western Philosophy and of Analytic Philosophy in particular. The publication of On Denoting by Bertrand Russell in that year gave rise to a very heated controversy – a true querelle philosophique – which has endured till now. There are two deployments: (a) on one side the Russell’s Description Theory vindicators like, among others, Carnap, Reichenbach, Quine, Blanché, (b) on the other its critics like Strawson, Searle, Geach, and, for several aspects, Kripke.

Whereas the former accept the Russelian thesis peacefully, without adding a new contribution, the latter examine it carefully in order to show its limits or to reduce it to its true proportions or even to confute and reject it. Even though I appreciate the brilliance of the Welsh philosopher, especially because his solution of problems arising from the use of denoting phrases allows a solution of puzzles such as – “the present King of France is bald”/ “the present King of France is not bald” – without rejecting the principle of excluded middle, I myself noted a great aporia concerning the relations between definite descriptions and identity after reading What is Identity? by C. J. F. Williams,\(^1\) and presented the results of such an enquiry at the 5th International Symposium “Logica ‘91” organised by the Czechoslovakian

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Academy of Sciences, Bechyne (Czechoslovakia as it was then), April 15-17, 1991. To the above aporia new difficulties, which are unsurmountable from a logical-analytical viewpoint, must be pointed out.

2. Description Theory Analysis by C. J. F. Williams

Before direct impact with Russell’s works, it is convenient to dwell on the pages by C. J. F. Williams on the Russellian individual or singular definite description theory. The scholar sets out various formulations of such a theory. He examines the above theory in four of Russell’s works: On Denoting (1905), Logic and Knowledge: Essays 1901-1950 (1956), Introduction to Mathematical Philosophy (1919) and Principia Mathematica (1910-1926).

Let us proceed in an orderly fashion.

As far as On Denoting is concerned Williams writes: «The meaning of the symbols of the form \((\forall x)(q\neg x)\) is given by contextual definition, by giving a paraphrase of complete sentences in which they occur. One such definition, which formalizes the general rule given in ‘On Denoting’, although it does not suffice for defining all occurrences of \((\forall x)(q\neg x)\), is

\[
(R) \psi ((\forall x)(q\neg x)) = \exists x (q\neg x \& (\forall y (qy \rightarrow x = y) \& q\neg x)).
\]

With \(q\neg x\) abbreviating ‘x wrote Waverley’ and \(q\neg x\) abbreviating ‘x was Scotch’ the definiens of the definition just given is the analysis Russell gives of ‘The author of Waverley was Scotch’. To get his analysis of ‘The author of Waverley is Scott’ (which is no different from ‘Scott is the author of Waverley’) in the formal version, all we need to do is to use \(a\) to symbolize ‘Scott’, so that

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we can substitute \( x = a \) (‘\( x \) is the same as \( a \)’) for \( qa \) in the *definiens*, to obtain

\[(\alpha) \exists x(qx \& [\forall y(qy \to x = y) \& x = a]).\]

This is obtained simply by following the rules by which Russell introduces his definite description symbol. \((\exists x)(qx) = a\), the symbolic version of ‘The author of *Waverley* is Scott’, is simply regarded as a definitional abbreviation of \((\alpha)\).\(^3\)

By \((\alpha)\) Williams deduces the following formulae:

\[(\beta) \exists x(qx \& x = a) \leftrightarrow qa\]

and

\[(\gamma) qa \& \forall y(qy \to a = y)\]

where \( qa \) stands for ‘Scott wrote *Waverley*’.

As far as *Logic and Knowledge* is concerned, Williams quotes the following passage: «It is not always false of \( x \) that \( x \) begat Charles II and that \( x \) was executed and that ‘if \( y \) begat Charles II, \( y \) is identical with \( x \)’ is always true of \( y \)».\(^4\) Then he gives the following translation in standard symbolism:

\[(A) \exists x[(qx \& \psi x) \& \forall y(qy \to x = y)].\]

It is clear that the author uses \( qx \) for ‘\( x \) begat Charles II’ and \( \psi x \) for ‘\( x \) was executed’.

As far as *Introduction to Mathematical Philosophy* is concerned Williams notes preliminarily that Russell gives as an analysis of ‘The author of *Waverley* was Scotch’ the conjunction of the three propositions: «\((B1) \ 'x \ wrote \ *Waverley*' \ is not always false ; \((B2) \ 'if \ x \ and \ y \ wrote \ *Waverley*, \ x \ and \ y \ are identical' \ is always true ; \((B3) \ 'if \ x \ wrote \ *Waverley*, \ x \ was Scotch’ \ is always

\(^3\) C. J. F. Williams, *What is Identity?*, cit. p. 12.

true». Then the scholar offers the following formulation always in standard symbolism:

\[(B) \exists x(qy) \& \{\forall y \forall z[(qy \& qz) \to y = z] \& \forall w(qw \to \psi w)\} \]  

It is evident that the scholar uses ‘\(q\)’ for ‘wrote Waverley’ and ‘\(\psi\)’ for ‘was Scotch’ as in (R) above, but changing the Russellian variables.

At last as far as Principia Mathematica is concerned Williams translates the Definition *14.01 in standard symbolism eliminating the definiendum and substituting the variable \(y\) for the sign \(b\):

\[(C) \exists x \forall y[(qy \leftrightarrow x = y) \& \psi y] \]  

«The difference between (A), (B), and (C) – the author concludes – lies in their length and in their perspicuity, or that of their ordinary language equivalents».

Note that Williams does not exhibit an instance concerning (C). Evidently he supposes the reader knows PM. Russell deals with Descriptions in Chapter III of the Introduction ⁷ and in *14.⁸ In a passage of the Introduction the British philosopher writes «we must not attempt to define “(\(\forall x\))(q(x))”, but must define the uses of this symbol, i.e. the propositions in whose symbolic expression it occurs. Now in seeing to define the uses of this symbol, it is important to observe the import of propositions in which it occurs. Take as an illustration: “The author of Waverley was a poet.” This implies (1) that Waverley was written, (2) that it was written by one man, and not in collaboration, (3) that the one man who wrote it was a poet».⁹ It is evident that in Williams’ formula (C) we can

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⁸ PM, pp. 173-186.
⁹ PM, pp. 67-68.
interpret ‘φ’ as “was author of Waverley” and ‘ψ’ as “was a poet”.

3. Equivocal use of predicates and individual variables in Williams’ formulae

What shocks the reader more is the coolness, or rather the unscrupulousness, with which Williams manipulates not only the predicates – Russell does this also – but even the individual variables. The analysis of

(R)  \( \psi ((\forall x)(\phi x)) = \exists x \{ qx \land [ \forall y (\phi y \rightarrow x = y) \land \psi x] \} \)

shows that this formula, just as it is written, is rather a contextual definition in the use of names par excellence since ‘φ’ and ‘ψ’ are monadic predicate in context. In fact using ‘φ’ for ‘is philosopher’ and ‘ψ’ for ‘is Greek’, we obtain the following interpretation: ‘The Philosopher is Greek’ is equal by definition to ‘There exists at least a \( x \) such that \( x \) is philosopher, and for every \( y \) if \( y \) is philosopher then \( x \) is identical with \( y \), and \( x \) is Greek’.

Now whereas the definiendum,

\( \psi ((\forall x)(\phi x)) \)

which asserts ‘The Philosopher is Greek’, is true, because ‘The Philosopher’ denotes Aristotle, the definiens

\( \exists x \{ qx \land [ \neg \exists y (qy \land x \neq y) \land \psi x] \} \)

is false, because it asserts that there exists one and only one philosopher and this is Greek. There is a disagreement between the definiendum, which asserts ‘The Philosopher is Greek’, and the definiens, which asserts ‘There exists one and only one philosopher and this is Greek’.

The difficulty is more evident if we consider Williams’ above formula...
(γ) \( qa \land \forall y (qy \rightarrow a = y) \)

which, by elementary rules of logic, is equal to

(γ .1) \( qa \land \neg \exists y (qy \land a \neq y) \).

Now, if we put ‘a’ for ‘Aristotle’, (γ) asserts ‘Aristotle is a philosopher and every philosopher is identical to him’, and (γ .1) asserts ‘Aristotle is a philosopher and there is not one philosopher different from him’. In other words the meaning of both – (γ) and (γ .1) – is the same: ‘Aristotle is the unique philosopher’ and this is a false statement.

A suspicion arises: in the case of names par excellence the singularising function is already hidden in the predicate of the definiens and does not emerge in a superficial, but only in a profound analysis of language, which has the courage to excavate the words to extreme limits.

The situation does not improve if we interpret ‘q’ as a dyadic predicate, as Russell does usually. Let ‘q’ stand for ‘the author of Perâ f›sewj’ and ‘p’ for ‘is Greek’. The above (R) will be read in the following way: ‘The author of Perâ f›sewj is Greek’ is equal by definition to ‘There is one x at least who is author of Perâ f›sewj, and for every y if y is author of Perâ f›sewj then x is the same as y, and x is Greek’. In this case both definiendum and definiens are false because there were several Greek philosophers (Anaximander, Anaximenes, Heraclitus, Parmenides), who wrote a work called Perâ f›sewj.

A new suspicion arises: the descriptive function has to contain inside itself at least one singularising element. This and/or these elements cannot be settled superficially by means of a monadic predicate like ‘q’, but they demand a contextual analysis in each case because these are very different between themselves. One requests not only the uniqueness of the argument which satisfies a given descriptive function, but also the singleness of the same function and such a uniqueness is not provided by a proper name, which can be given to several different individuals, but by the Peanian inverted iota-operator which is hidden in the proper
name and which does non emerge in the course of a superficial but only in the course of a profound analysis of language. In other words a rigorous language analysis requires two conditions in the case of a dyadic predicate of a descriptive function: (a) the uniqueness of the individual variable which satisfies the function and which is its first argument of such a function, and (b) the uniqueness of the second argument already belonging to the function and which is its integrand part.

Indeed, if we consider carefully, this latter is the singularising element which entitles to assign the function not only to one and only one individual but also to that and not to other individuals. In the case the descriptive function is made of a dyadic predicate whose second argument, which is immanent in it, is a proper name, a rigorous language analysis requires not only such a function cannot be settled by a monadic predicate like ‘φ’, but also such a function must be analysed in its singularising components, especially if there is an equivocal use of proper names. Unfortunately the analytical philosophers often are half analytical. *Usque ad finem dispicere aude!* This must be the program of every rigorous analytical philosopher. You must have the courage to be analytical philosopher to the end, without stopping half way.

All the observations above made regarding the transcription of Russellian definite singular description theory in *On Denoting*, labelled by Williams with (R), hold also for the transcription of the same theory in *Logic and Knowledge*, *Introduction to Mathematical Philosophy* and *PM*, labelled by Williams with (A), (B) and (C) respectively.

Only a particular consideration has to be made regarding (B) where we observe a true and evident violation of the Ockham’s razor “entia non sunt multiplicanda praeter necessitatem” or also “frustra fit per plura quod fieri potest per pauciora”. Let us come back to (B)

(B) \( \exists x(q_y) \& \{ \forall y \forall z[(q_y \& q_z) \rightarrow y = z] \& \forall w(q_w \rightarrow \psi_w) \}. \)
This expression is very strange. In fact either this formula is without meaning or there is a mistake. I think that there is a typographical oversight. In order to save the sense of Williams’ expression let us rectify the above transcription of (B) as follows:

\[(B.\text{bis})\quad \exists x(qx) \land \{\forall y \forall z[(qy \land qz) \rightarrow y = z] \land \forall w(q\psi w \rightarrow \psi w)\}.
\]

Nevertheless the perplexities last. Even with the above correction it is to difficult for us to understand why in (B) the variables, which on the base of Russelian text \(^{10}\) should be two, \(x\) and \(y\), as follows

\[(B.\text{ter})\quad \exists x(qx) \land \{\forall x \forall y[(qx \land qy) \rightarrow x = y] \land \forall x(qx \rightarrow \psi x)\}
\]

become four: ‘\(x\)’, ‘\(y\)’, ‘\(z\)’ and ‘\(w\)’. Now either the two latter variables are superfluous, since they always range over the same values, or Williams has realised there are troubles when we translate the universal affirmative sentences in the negation of the corresponding particular negative ones.

4. A trend to the symbolic rigour: Reichenbach and Blanché

We find a rigour unknown to Williams, and before him to Russell self, in the works by Reichenbach and Blanché.

Hans Reichenbach, after having asserted that he uses the Peanian and Russellian notation

\[(\exists x)f(x)\]  \hspace{1cm} (1)

to say ‘the thing \(x\) having the property \(f\)’, distinguishes two classes of arguments: on one side persons and things, on the other space-time determinations.

\(^{10}\) «(B1) ‘\(x\) wrote Waverley’ is not always false ; (B2) ‘if \(x\) and \(y\) wrote Waverley, \(x\) and \(y\) are identical’ is always true; (B3) ‘if \(x\) wrote Waverley, \(x\) was Scotch’ is always true», B. Russell, Introduction to Mathematical Philosophy, London, George Allen and Unwin, 1919, p. 177. See C. J. F. Williams, Op. cit., p. 15.
As far as the first class of arguments is concerned the German philosopher writes: «Statements containing a proper name, and statements containing a description, are alike in so far as they tell us that a proper name is introduced by definition as a sign of the thing, whereas if we apply a description to a thing we must ask whether the application is correct. Thus the statement ‘London is called ‘London’’ tell us only that there is a thing having that name; the rest is tautologous by definition. Contrarily, the statement ‘London is the capital of England’ not only tells us that there is a thing named ‘London’ but also adds synthetic information about the thing. The word ‘is’ in this example expresses the relation of identity. Thus the sentence is symbolized by

\[ x_1 = (\exists x)c(x, z_1) \]  

where ‘\(x_1\)’ means ‘London’, ‘\(z_1\)’ means England, and ‘\(c\)’ means ‘capital’».\(^{11}\)

In spite of the fact that in metalanguage the inverted commas of word ‘England’ have be omitted by a typographical mistake, it is clear that in formula (2) the signs ‘\(x_1\)’, ‘\(c\)’ and ‘\(z_1\)’ are used in formal, not in material supposition.

As far as the second class of arguments is concerned the German philosopher observes: «The use of space-time indications is an important means of making descriptions in things unambiguous. Thus papers of identification, like passports, show place and time of birth of persons, in addition to the name. This kind of description is used even for the introduction of proper names of persons. In the birth registers we find statements of the form statement ‘the child born to Mrs. N on June 17, 1917, receives the name ‘Isabelle’’. This statement is symbolized by

\[ x_i = (\exists \alpha)b(x, z_1, t_1) \]  

where ‘b’ means ‘born’. Here the sign = stands between arguments, not between propositions».\(^{12}\) Also in this case ‘x’, ‘b’, ‘z’ and ‘t’ are used in formal and not in material supposition. The meaning of all expression, is “Isabel (i.e. the x named ‘Isabel’) is equal by definition to the x such that was born to Mrs. N (z) on June 17, 1917 (t)”.\(^{13}\)

A very important passage follows that is convenient to quote in its entirety: «Let us start from the example ‘George VI was crowned at Westminster Abbey’, which we symbolize by

\[
c(x_1, y_1)
\]

where ‘c’ means ‘crowned’, ‘x_1’ means ‘George VI’, and ‘y_1’ means ‘Westminster Abbey’. For ‘George VI’ we can put ‘the king of England’; i.e., we have the statement

\[
x_1 = (\forall x)k(x, z_1)
\]

where ‘k’ means ‘king’ and ‘z_1’ means ‘England’. Putting (5) in (4) we have

\[
c \left[ (\forall x)k(x, z_1), y_1 \right]
\]

for the statement ‘the king of England was crowned at Westminster Abbey’. We see that a sentence with a description as argument has a structure similar to that of a sentence with a proper name as argument, with the difference, however, that the argument itself has an inner structure; it is a compound term including a propositional function, other arguments, and a bound variable.

When we are asked to give a definition of the iota-operator we must say, that we cannot define a term like (1), but that we can give a definition for every sentence in which the iota operator occurs. That is, we have only a definition in use for the iota-operator. Thus we define the sentence (6) by the sentence

\(^{13}\) Op. cit. loc. cit.
\[ \exists x \{ k(x, z_1) \cdot c(x, y_1) \cdot (u) [k(u, z_1) \supset (u = x)] \} \]  

(7)

We see that a sentence containing a description can be replaced by an existential sentence which includes a qualification that there is only one argument satisfying the function; this qualification is expressed in the last term of (7) stating that all things \( u \) satisfying the function \( k \) are identical with \( x \). The notation (6) corresponds exactly to the form of speaking used in conversational language; (7) is a transcription in terms of an existential operator.\(^{14}\)

It is a matter of an analysis very exemplar as rigour is concerned. It is a pity that there is a mistake inside the definiens (7). This later is an existential declarative sentence whose scope is a logical product of three conjuncts

\[
\begin{align*}
(a.1) & \quad k(x, z_1) \\
(a.2) & \quad c(x, y_1) \\
(a.3) & \quad (u) [k(u, z_1) \supset (u = x)].
\end{align*}
\]

Now the first conjunct is a descriptive function which does denote George VI. In fact the definiens asserts that there is at least a \( x \) such that \( x \) is king of England and \( x \) was crowned at Westminster and for every \( u \) if \( u \) is king of England then \( u \) is the same as \( x \), i.e. there is one and only one king of England and he was crowned at Westminster, but such denoting phrase cannot refer to the only George VI because all kings of England are crowned at Westminster. In this case Reichenbach, who is always careful to consider space-temporal parameters, has not exhibited a descriptive function to individualise as the sole denoted individual George VI.

Inaccuracies of such a sort, which appear already in Russell’s On Denoting, have been focused by Peter Frederick Strawson, who in his authoritative paper On Referring \(^{15}\) has emphasised the language pragmatic dimension. Do not forget that it is a matter, once again, of the philosophical individuation


problem, which so much strove the mediaeval thinkers together with that of unity or plurality of forms from one side and that of universals on the other. It is not a case that ‘the present king of France is wise’ has different truth values if uttered by a contemporary of Louis XIV or of Louis XV.

I think you must chose between two alternatives: either you insert individuating parameters inside the descriptive function – in the Reichenbach’s instance (7) the temporal ones are sufficient since those concerning the space have already been supplied – or the language pragmatic dimension cannot be supplanted. Do not forget besides that even in the cases the only syntactic and semantic dimensions leave aside the pragmatic one – this is e.g. the case of a scientific ideal language – it is always a matter of a set of signs combined by rules common to a speaking community, therefore always guided by intentional speech acts of speaker and always directed to receptive and interpretative capacities of the listener.

A scholar who offers a symbolism absolutely void of ambiguity in matter concerning the theory of definite singular description is the French scholar Robert Blanché, whom the following analysis is due.

Let the sentence “Voltaire is a great prose-writer” be. Instead of “Voltaire” we may use the locution “The author of Candide” in order to denote the same person. Now let \( f_{xy} \) stand for “author of Candide”, \( (\forall x) f_{xy} \)' for “the \( x \) who is author of Candide” and \( g((\forall x) f_{xy}) \)' for “The author of Candide is a great prose-writer”. When we employ a descriptive expression, we presuppose both: 1) the existence of one individual corresponding to this description; 2) the unicity of such an individual. Let us sketch the symbolic passages: first presupposition; second presupposition; final assertion:

\[
(\exists x) \cdot f_{xy},
\]

\[
(\exists x) \cdot f_{xy}, : (z) \cdot f_{zy} \supset (z = x)
\]

\[
(\exists x) \cdot f_{xy}, : (z) \cdot f_{zy}, \supset (z = x) : gx
\]
The sentence “The author of Candide is a great prose-writer” is analysed as follows “There exists one \( x \): 1) who is author of Candide, 2) such that, anybody be who you designate as author of Candide, will be identical to \( x \), 3) such that this \( x \) is a great prose-writer.”. Let us conclude here all three conditions: existence, uniqueness and subsumption – to use the precise and rigorous terminology by Nathan Salmon – are satisfied. Here all equivocations are disappeared since all terms are been made explicit.

It would appear that all difficulties are eliminated by means of Blanché’s explanations. On the contrary those peep through the theses of the most celebrated logical work of the first twenty sex years of Twentieth Century.

5. Troubles in PM and devices for their overcoming

True troubles begin with PM. Let us start from the definition

\[ \land \:
(\forall x) (\forall y) : (x = y) \iff \land (x)(y) = a \\

and from two propositions

\[ \land \:
(\forall x) (\forall y) : (x = y) \iff \land (x)(y) = a \\

and

\[ \land \:
(\forall x) (\forall y) : (x = y) \iff \land (x)(y) = a \\

Now let us consider the first atomic sentence of the equivalence *14.13, that is to say:


\((\forall x)(p\cdot x) = (\forall x)(q\cdot x)\)

where ‘(\forall x)(p\cdot x)’ is the argument and ‘(\forall x)(q\cdot x)’ is the sentential function which is satisfied by that.

Let us mechanically substitute in Definition *14.01 the monadic predicate ‘p’ with the sentential function ‘(\forall x)(q\cdot x)’, which we write temporarily before the corresponding argument as Łukasiewicz does, but inserting it in heavy type braces in order to avoid confusion

\[(\forall x)(p\cdot x)\]
\[\{ = (\forall x)(q\cdot x)\} (\forall x)(q\cdot x) \equiv \exists b : q\cdot x. \equiv x = b: \{ = (\forall x)(q\cdot x)\} b\] Df

Now writing the identity predicate between the corresponding terms, as usual in every symbolism different from the Polish one, we obtain

\[(\forall x)(p\cdot x)\]
\[(\forall x)(p\cdot x) = (\forall x)(q\cdot x) \equiv \exists b : q\cdot x. \equiv x = b: b = (\forall x)(q\cdot x)\] Df.

As you can see, in this case the inverted iota-operator reappears in the definiens, and cannot be eliminated by a contextual definition.

The same takes place if instead of the first sentence of the equivalence *14.131

\((\forall x)(p\cdot x) = (\forall x)(q\cdot x)\)

we examine the second one

\((\forall x)(q\cdot x) = (\forall x)(p\cdot x)\).

In order to avoid such a trouble, which would produce the collapse of Russellian Description Theory, the authors of *PM* prove theses *14.13 and *14.131 as follows:

*14.13*  \( \vdash a = (\forall x)(p\cdot x). \equiv (\forall x)(q\cdot x) = a \)

Dem.
At first sight it would appear that both dangers are eliminated for ever, i.e. (1) the apparition of the Peanian inverted iota operator inside the *definiens*, (2) the risk of its being presupposed as a primitive locution exactly as proper nouns. But we ask: is this how things are? In order to answer let us come back to Williams’ analyses.

**6. Williamsian analysis of formulae of types** \( \forall x (\varphi x) \) and \( (\forall x)(\varphi x) = (\forall x)(\varphi x) \)

William, who agrees to Wittgenstein’s viewpoint that identity is not a relation, not only wishes to introduce the Peanian
inverted iota-operator by means a contextual definition, as Russell
does, but also to eliminate the identity sign by means of such a
definition, as Russell does not. He wants simplify both: formulae
of type
\[ a = (\forall x)(\varphi x) \]
and those of type
\[ (\forall x)(\varphi x) = (\forall x)(\psi x). \]

**Simplification of formulae of type** \[ a = (\forall x)(\varphi x) \].

Let us come back to Williams’ formulae (A), (B) and (C). The scholar writes: «Another formula, which is logically
equivalent to (A), (B) and (C) is
\[ (D) \ exists x (\exists x \land y) \land \neg \exists y [\exists z (y \land z) \land y \neq z]. \]

This gives the form of the proposition ‘Someone wrote *Waverley*
and was Scotch and no two people wrote *Waverley*. Any
proposition of the form \((\exists x)(\varphi x)\) is equivalent to one of this
form. In particular, the pattern provided by (D) will provide the
analysis of a proposition of the form \(a = (\forall x)(\varphi x)\) by the simple
technique of substituting \(a = x\) for \(\exists x\). We thus obtain
\[ (E) \ exists x (\exists x \land a = x) \land \neg \exists y [\exists z (y \land z) \land y \neq z]. \]

The first conjunct of this, \(\exists x (\exists x \land a = x)\), is easily seen be a mere
periphrasis for \(qa\), since (β), i.e., \(\exists x (\exists x \land a = x) \leftrightarrow qa\), is
logically true: ‘Someone identical with Scott wrote *Waverley’ is
just another way of saying ‘Scott wrote *Waverley*. That (β) is a
logical truth is [...] something of which Russell was aware, and
which he made use of in abbreviating (α) to (γ). Russell derives
the equivalence of these formulae, i.e.
\[ \exists x (\exists x \land \forall y (\varphi y \rightarrow x = y)) \land x = a \leftrightarrow [qa \land \forall y (\varphi y \rightarrow a = y)], \]
from (β) by substituting \( \phi x \& \forall y (qy \& x = y) \) for \( \phi x \) in (β). Wittgenstein’s abbreviation uses (β) itself to abbreviate the first conjunct of (E). This gives, as a shorter formula logically equivalent to to (E),

\[(F) \quad \phi a \& \neg \exists x \exists y [(qx \& qy) \& x \neq y].\]

That is to say, the Theory of Descriptions gives ‘Scott wrote Waverley and no two people wrote Waverley’ as the analysis of ‘Scott was the author of Waverley’\(^{18}\).

Therefore for Williams

\[a = (\forall x) (\phi x)\]

is the same as (F)

\[\phi a \& \neg \exists x \exists y [(qx \& qy) \& x \neq y]\]

where neither the Peanian inverted iota-operator nor the identity symbol appear.

**Simplification of formulae of type**\( (\forall x) (\phi x) = (\forall x) (\psi x) \)

Once again Williams takes (D) as the model for analysing sentences of type \( (\forall x) (\phi x) = (\forall x) (\psi x) \).

He writes «First we reletter the variables to avoid confusion with variables already used in (D), replacing the formula \( (\forall x) (\phi x) = (\forall y) (\psi y) \) with \( (\forall w) (\psi v) = (\forall w) (\phi w) \). Then, substituting \((\forall v)(\psi v) = x\) for \(qx\) in (D), we obtain

\[(G) \quad \exists x (qx \& (\forall v)(\psi v) = x) \& \neg \exists y \exists z [(qy \& qz) \& y \neq z].\]
Once again the first conjunct of (G), \( \exists x(qx \land (\forall y)(\psi y) = x) \), can be abbreviated to \( \varphi((\forall y)(\psi y)) \): ‘Someone identical with the author of Marmion wrote Waverley’ is just another way of saying ‘The author of Marmion wrote Waverley’. So (G) may be abbreviated to

\[(H) \quad \varphi((\forall y)(\psi y)) \land \neg \exists y\exists z((\varphi y \land \varphi z) \land y \neq z).\]

Now it is a simple matter to replace \( \varphi((\forall y)(\psi y)) \) as it occurs in (H) by its \textit{definiens} according to the Theory of Descriptions, namely \( \exists x(qx \land qx) \land \neg \exists w\exists v[(\varphi w \land \varphi v) \land w \neq v] \), which is (D) with \( w \) and \( v \) substituted for \( y \) and \( z \) and \( \varphi \) swapped with \( \psi \). We thus obtain

\[(I) \quad \exists x(qx \land qx) \land \{\neg \exists w\exists v[(\varphi w \land \varphi v) \land w \neq v] \land \neg \exists y\exists z((\varphi y \land \varphi z) \land y \neq z}\}.\]

Merely to restore the normal order of variables, we then exchange the second and the third conjuncts and the subordinate conjuncts of the matrix of the first conjunct of (I), to obtain

\[(J) \quad \exists x(qx \land qx) \land \{\neg \exists y\exists z((\varphi y \land \varphi z) \land y \neq z) \land \neg \exists w\exists v[(\varphi w \land \varphi v) \land w \neq v]\}.\]

That is to say, the Theory of Descriptions gives ‘Someone wrote both Waverley and Marmion, and no two people wrote Waverley, and no two people wrote Marmion’ as the analysis of ‘The author of Waverley was the author of Marmion’. This is second simplification, the simplification of the second form of identity proposition allowed by Russell, namely, that which contains two definite descriptions».\(^19\)

7. \textbf{Difficulties which emerge from the revision by Williams of the formulae of type} \((\forall x)(\varphi x) = (\forall x)(\psi x)\)

It is no use your repeating the above criticism concerning the unscrupulous employment of ‘$\varphi$’ and ‘$\psi$’ made by Williams. Whereas these letters in the above contexts are monadic sentential functions – Geach would have said ‘monadic predicables’ – they are interpreted by Williams as dyadic sentential functions, whose second argument always are proper names. Proper names, mind you! Let us begin from the analysis of the second point which concerns the reduction of the formula

$$(\forall x)(\varphi \cdot x) = (\forall x)(\psi \cdot x)$$

which is the first sentence of proposition *14.131 of PM to the following

$$(J) \exists x(qx \& qx) \& \{\neg \exists y \exists z[(qy \& qz) \& y \neq z]$$
$$\& \neg \exists w \exists v[(qw \& qv) \& w \neq v]\}.$$ 

In Williams’ opinion, the later formula, contrarily to the first, would allow the elimination of both: the identity sign and the inverted iota operator. Now let us take three instances: 1. The author of *Il barbiere di Siviglia* (The barber of Seville) is identical with the author of *La gazza ladra* (The thief magpie); 2. The author of *Turandot* is identical with the author of *Madama Butterfly*; 3. The author of *Macbeth* is identical with the author of *Hamlet*. Since (J) is a logical product of three sentences, let us dismember them:

(i) $\exists x(qx \& qx)
(ii) \neg \exists y \exists z[(qy \& qz) \& y \neq z]
(iii) \neg \exists w \exists v[(qw \& qv) \& w \neq v]

In the first case if we interpret ‘$\varphi$’ as ‘is author of *Il barbiere di Siviglia*’, ‘$\psi$’ as ‘is author of *La gazza ladra*’ we shall have: (i) there is at least one $x$ who is author of *Il barbiere di Siviglia* and of *La gazza ladra*; (ii) do not exist two different
authors of *Il barbiere di Siviglia*; (iii) do not exist two different authors of *La gazza ladra*.

In the second case if we interpret ‘φ’ as ‘is author of *Turandot*’, ‘ψ’ as ‘is author of *Madama Butterfly*’, we shall have: (i) there is at least one \( x \) who is author of *Turandot* and of *Madama Butterfly*; (ii) do not exist two different authors of *Turandot*; (iii) do not exist two different authors of *Madama Butterfly*.

In the third case if we interpret ‘φ’ as ‘is author of *Macbeth*’, ‘ψ’ as ‘is author of *Hamlet*’, we shall have: (i) there is at least a \( x \) who is author of *Macbeth* and of *Hamlet*; (ii) do not exist two different authors of *Macbeth*; (iii) do not exist two different authors of *Hamlet*.

Now (ii) is false in all three interpretations. In fact in the first case there exist two different individuals who are authors of *Il barbiere di Siviglia*: Gioacchino Rossini (Pesaro 1792- Passay near Paris 1868) and Giovanni Paisiello (Taranto 1740-Naples 1816); in the second case there exist two different individuals who are authors of *Turandot*: Giacomo Puccini (Lucca 1858-Brussels 1929) and Ferruccio Busoni (Empoli 1866-Berlin 1924); in the third case there exist two different individuals who are authors of *Macbeth*: William Shakespeare (Stratford-on-Avon 1564-1616) and Giuseppe Verdi (Busseto 1813-Milan 1901). Since in a logical product even also only one of its logical conjuncts which is false falsifies all molecular sentence, all three above sentences result be false. The point is that there are two *Il barbiere di Siviglia*, two *Turandot*, two *Macbeth*, different between them, and therefore the singularising function, expressed by Peano and Russell with the sign ‘\( \gamma \)’ and by Frege by the sign ‘\( \dot{\gamma} \)’ ‘\( \ddot{\gamma} \)’, which individualises not only one and only one individual but just that and no other, must intentionally be immanent in such names. Rossini is author of that *Il barbiere di Siviglia* very different from all other *Il barbiere di Siviglia*; Puccini is author of that *Turandot* very different from all other *Turandot*; Shakespeare is author of that *Macbeth*, very different from all other *Macbeth*. Nothing is so presupposed in the singular definite descriptions as the singularising function.
The affairs do not change for the better if we come back from Williams’ formulae to the four Russellian singular definite description contextual definitions. Let the following three instances be:

The author of *Il barbiere di Siviglia* is Italian;
The author of *Turandot* is Tuscan;
The autor of *Macbeth* is European.

Let us start from the contextual definition of *On Denoting*, labelled by Williams with (R)

\[ \psi ((\forall x)(\varphi x)) = \exists x\{\varphi x \& [\forall y(\varphi y \rightarrow x = y) \& \psi x]\} \]

which can be written as follows on the base of elementary logical rules

(R.bis) \( \psi ((\forall x)(\varphi x)) = \exists x\{\varphi x \& [\exists y(\varphi y \& x \neq y) \& \psi x]\} \).

The complex sentential function, which appears inside the scope of the existential quantifier of the *definiens*, is a logical product of two conjuncts:

(j) \( \varphi x \)

(jj) \( \neg [\exists y(\varphi y \& x \neq y) \& \psi x] \).

This later is in turn a logical product made up two conjuncts

(jj.bis) \( \neg [\exists y(\varphi y \& x \neq y) \& \psi x] \)

(jj.ter) \( \psi x \).

In the first case interpreting ‘\( \varphi \)’ with ‘is author of *Il barbiere di Siviglia*’ and ‘\( \psi \)’ with ‘is Italian’ we obtain ‘The author of *Il barbiere di Siviglia* is Italian’ = \( \exists x \) ‘There is at least one \( x \), such that (j) \( x \) is author of *Il barbiere di Siviglia*, and (jj.bis)
there is not one author of *Il barbiere di Siviglia* different from \( x \), and (jj.ter) \( x \) is Italian’.

In the second case interpreting ‘\( \varphi \)’ with ‘is author of *Turandot*’ and ‘\( \psi \)’ with ‘is Italian’ we obtain ‘The author of *Turandot* is Tuscan’ = \( \exists y \ (y \text{ is author of } *Turandot* \land y \text{ is Tuscan}) \).’

In the third case interpreting ‘\( \varphi \)’ with ‘is author of *Machbeth*’ and ‘\( \psi \)’ with ‘is European’ we obtain ‘The author of *Machbeth* is European’ = \( \exists y \ (y \text{ is author of } *Machbeth* \land y \text{ is European}) \).

The above three definitions do not individualise Rossini, Puccini and Shakespeare, since (jj.bis) is false in all three instances. In fact there is a second author of *Il barbiere di Siviglia* different from Rossini who satisfies the property of being Italian, there is a second author of *Turandot* different from Puccini who satisfies the property of being Tuscan, there is one other author of *Machbeth* different from Shakespeare who satisfies the property of being European.

In this cases the proper name which is an integral part of descriptive function is not sufficient to individualise the subject of the definite description. We need the Peanian inverted iota operator which thrown out gets in by the back-door. In fact we are compelled to say in the first case ‘There is at least one \( x \), such that (j) \( x \) is author of that *Il barbiere di Siviglia*, which has the properties so and so, and (jj.bis) there is not one author of that *Il barbiere di Siviglia*, which has the properties so and so, different from \( x \), and (jj.ter) \( x \) is Italian’. In the second case we are compelled to say ‘There is at least one \( x \), such that (j) \( x \) is author of that *Turandot*, which has the properties so and so, and (jj.bis) there is not one author of that *Turandot*, which has the properties so and so, different from \( x \), and (jj.ter) \( x \) is Tuscan’. In the third case we are compelled to say ‘There is at least one \( x \), such that (j) \( x \) is author of that *Machbeth*, which has the properties so and so,
and (jj.bis) there is not one author of that *Machbeth*, which has the properties so and so, different from *x*, and (jj.ter) *x* is European.’

Things do not change if we chose the other Russell’s definitions labelled by Williams with (A), (B) and (C) and we give the same interpretations to predicate ‘*ψ*’ e ‘*ψ*’.

What we shall say when there is one and only one proper name, which is given to one and only one single person or thing without risk of misunderstanding? Even in this case the use of Peanian inverted operator is presupposed as Reichenbach noted without drawing the consequences logically. In fact is it which allows the definition of a proper name, which in turn is introduced in a formula without a contextual definition.

8. Conclusion

Given the Russell’s definition (R)

\[ \psi ((\forall)(\phi x)) =_w \exists x(\phi x \& [\forall y(\phi y \rightarrow x = y) \& \psi x]) \]

or one other equivalent among those analysed above by Williams, either we interpret ‘*ψ*’ as a monadic predicate or as dyadic one.

If we interpret ‘*ψ*’ as a monadic predicate, then we obtain the definition of names par excellence, but such a definition does not work as we have seen above at § 3. In this case the Peanian inverted iota operator is presupposed and cannot be introduced by means a contextual definition.

If we interpret ‘*ψ*’ as a dyadic predicate it requires a proper name as second argument of the descriptive function, which in turn must be satisfied by the *x* whose existence and uniqueness are asserted. Now either such a proper name is given to one and only one person or thing, without the risk of misunderstanding, or to several. If a proper name is given to one and only one person or thing without a risk of misunderstanding, then it denotes not only one and only one individual but that one and not one other, and therefore the Peanian inverted iota operator is presupposed by the function which would introduce it by means a contextual
definition. If a proper name is given to several things or persons with the risk of misunderstanding, then the Peanian inverted iota operator is requested all the more reason in order to individualise the only individual to whom or to which such an equivocal proper name is given.

On such bases we are compelled to conclude that Russell’s definition of definite individual description is circular since it presupposes the use of Peanian inverted iota operator which it ought introduce contextually. Consequently the Russelian definition, important in order to focalise a set of problems but inadequate to solve them, must be rejected. It is, to use a Wittgenstein’s image, the useful ladder that must be thrown away after the philosopher has climbed up it.

**Selected Bibliography**


