The Relevance of Paraconsistent Logic in Experimental, Theoretical and Foundational Contexts

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ABSTRACT. I will present paraconsistent logic in three different contexts: i) the experimental context, in which a statement is considered as concerning an experimental datum (Vasiliev); ii) the theoretical context, in which a statement is considered in a provisional way, as a candidate for playing inside a theoretical framework the role of a principle; iii) the foundational context, where a same statement leads to compare two different semantics according to two different formalizations of a same theory; in other terms, when these formalizations are mutually incommensurable. The case-study of the original Goe-del's proof is analysed as an instance of the last use of paraconsistent logic.

1. The occurrence of double negated statements inside scientific theories

The viewpoint of present paper is to consider the kind of logic as determined by the kind of the organisation of a scientific theory. Usually the Aristotelian kind of organisation, i.e. the entirely deductive one, is taken in account. Instead, in the history of science there exist several theories whose organisation is aimed to search for a new scientific method, capable to solve an universal problem: e.g., classical chemistry, L. Carnot's theories on geometry, calculus and mechanics, S. Carnot's thermodynamics, Lobachevsky's non-
Euclidean geometry, Galois's theory, Klein's Erlanger program, computability theory and Bourbaki's program (whose problem is how much structures constitute the basis to Mathematics).

To these two kinds of organisations correspond respectively classical logic and intuitionistic logic; in an original text the latter one is recognised by the occurrence of some double negated sentences, each one of which is not equivalent to the corresponding positive sentence, owing to its lack of scientific evidence (= DNS); e.g. Lobachevsky never wrote the positive statement: "There exist two parallel lines"; always he wrote a DNS, e.g.: "The second assumption [of two parallels] can likewise be admitted without leading to any contradiction in the results...".¹ This feature constitutes a failure of the double negation law; hence, the original presentation of the theory by the author followed a non-classical logic, at least the intuitionistic one.

This dichotomic option on the kind of organisation (and at the same time on the kind of logic too) can be traced back to Leibniz' philosophy of science; he suggested two basic logico-philosophical principles as underlying our theoretical activity, i.e. the non-contradiction principle and the principle of sufficient reason, itself stated by means of a DNS: "Nothing is without reason, or everything has its reason, although not always we are capable to find out this reason...".²

2. Experimental context and Vasiliev's paraconsistent logic

The founder of paraconsistent logic, N.A. Vasiliev, stated as a characteristic feature of his logic, three kinds of sentence, i.e. “S is A”, “S is not A”, “S is and is not A”; the last one, i.e. the

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“indifferent judgement” is the characteristic sentence of his paraconsistent logic. A scholar illustrates this point in the following way:

"… Vasiliev… proceeds from the presumption that an experiment gives us directly only singular positive judgements. We have no sense organs for the observation of the absence of properties of objects.

Negative sentences are the result of deduction. Suppose that I say: “this book is red”. I have no way of observing directly the absence of red colour, but I see that this book is yellow. Knowing that the object cannot simultaneously be both red and yellow, I deduce from this observation and from my knowledge of the incompatibility of yellow and red colours, that this book is not red. The very incompatibility of red and yellow is, of course, an ontological characteristic of our world. In another world such incompatibility may not exist.

Now let us assume that the subject has the capacity of observing not only the presence but also the absence of a property… this… possibility of getting simultaneously knowledge about the absence and presence of a certain property depends on some external conditions. In Aristotelian logic a negative sentence coincides [just] with the assertion of the falsity of the positive sentence, and is essentially a complex sentence. In imaginary logic, a singular negative sentence has an independent character and does not coincide with the assertion of the falsity of the positive sentence… [Hence.] Instead of the two types of singular sentence - positive and negative - which may be compatible, it is possible to introduce three types of singular, atomic sentences - positive, negative, indifferent - which are pairwise inconsistent."

Let us add a further appraisal:

“It is easy to see that a logic dual in relation to intuitionistic logic is paraconsistent… If a logic dual in relation to the intuitionistic logic is considered as paraconsistent, and a relational semantic in Kripke is constructed for it, we shall see that it is based on the principle of...

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conservation of falsity. What is recognised to be false will never become true, but that which is accepted at a given moment may subsequently be rejected.”

This is just what an experimental scientist does.

We conclude that paraconsistent logic is a very old logic inside the minds of experimental scientists. In past times, this kind of logic was not recognised in this experimental context since the theoretical viewpoint was privileged; from a theoretical viewpoint the work done by an experimental scientist appears as an obscure work from which the theoretical scientist has to get the output result only, and then he idealises it in either an irrational number, or an idealised notion or an ideal law on the nature; and he idealizes logic too, as the classical one.

3. Formalisation of Vasiliev paraconsistent logic

In a previous paper\(^6\) I obtained a relevant result regarding paraconsistent logic. I showed that Vasiliev's three sentences hold true even when one substitutes "\(\neg\neg A\)" for "S", "\(\rightarrow\)" for "is" and "fails to \(\rightarrow\)" for "is not". One obtains respectively: "\(\neg\neg A \rightarrow A\)", "\(\neg\neg A \text{ fails to } \rightarrow A\)" "\(\neg\neg A \rightarrow A \text{ and } \neg\neg A \text{ fails to } \rightarrow A\)".

My interpretation of Vasiliev's logic is suitably represented by da Costa and Puga's formal clauses for Vasiliev's paraconsistent logic\(^7\): 1) To classical negation (\(\text{de dicto}\)) a weak negation (\(\text{de re}\)) \(~\) is added, such that (\(A \lor \sim A\)) and \((\neg(\neg A \sim A))\) are not theorems. 2) Vasiliev's metalogic (i.e. the invariable kernel of logical laws) is interpreted as classical logic; hence, it includes a negation \(~\) whose meaning is "false". 3) If the negation \(~\) behaves

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as \neg, then classical logic is obtained. 4) In propositional calculus weak negation applies to atomic formulas only; owing to this requirement the symbol of negation may be only one. 5) Subsequent theorems of da Costa and Puga's formal system, rather than Arruda's, agree with my interpretation of Vasiliev's paraconsistent logic.\footnote{A.I. Arruda: "Imaginary Logic in Vasil'év", in A.I. Arruda, N.C.A. da Costa, R. Chaqui (eds.): \textit{Non-Classical Logics, Model Theory and Computability}, North-Holland, 1977, \textit{3-24}. In particular, compare in p. 19, Th. 4.2 with Th. 8 by da Costa and Puga. D'Ottaviano's formalization of paraconsistent logic too is interesting; it introduces, beyond the values true and false, a provisional value too; the semantic of the last value agrees with the meaning I gives to Vasiliev's indifferent judgement; unfortunately, D'Ottaviano's formalization chooses the classical law of double negation. See R.L. Epstein: \textit{The Semantic Foundations of Logic}, vol. 1, Kluwer Acad. P., 1995, 265-287.}

4. Theoretical context and Paraconsistent logic

"The decisive fact in understanding the categorical judgements in Vasiliev is the division of judgements into factual ones and judgements about notions. Judgements on facts are judgements stating the results of observations or experiments. Judgements on notions are judgements expressing laws, or nomological statements in modern terminology."\footnote{V.A. Smirnov: "The logical …", p. 628.}

The quotation supports the view that knowledge from nature is not obtained by mere hard facts; indeed, the search for an interpretative law arises up from a deliberate asking nature by means of a hypothesis; which has to be considered as a mere guess before it is tested against nature; it becomes a methodological principle when it is corroborated by some facts; it achieves the status of a law when a lot of facts consistently confirms previous guess.
In this context paraconsistent logic is unrecognized because an experimental scientist each time advances a single guess; and the theoretical role of a guess was apperceived by him in an obscure way. Moreover, no one scientist obtained a consistent arguing according to paraconsistent logic, that is no one produced a scientific theory by linking a guess together with some other guesses; since he was aware that this possible construction constitutes rather a metaphysical philosophy, which could become a scientific theory only after a long work for successfully supporting each its sentence on hard facts.

When searching for a comprehensive theoretical framework, a theoretical scientist exploits a lot of experimental sentences as his hard data, to be collected together according to some principles which not necessarily are experimental in nature. When a theoretical scientist leaves a sentence - not entirely experimental in nature - in a provisional state with respect to its possible qualification as either an axiom-principle or a methodological hypothesis, he consistently makes use of paraconsistent logic.

Indeed let us remark that the above three Vasiliev's sentences respectively represent the three different roles which can be played by a sentence in a theoretical context in progress.

\( i \) \( \neg \neg A \rightarrow A \) represents an affirmative sentence, i.e. a sentence supported by sufficient scientific evidence;

\( ii \) \( \neg \neg A \text{ fails to} \rightarrow A \) represents a problem, i.e. a sentence as yet insufficiently supported by scientific evidence;

\( iii \) \( \neg \neg A \rightarrow A \text{ and} \neg \neg A \text{ fails to} \rightarrow A \) represents a sentence whose truth or falsity is as yet undecided in scientific terms; inside a theoretical framework this kind of sentence may be considered as a guess, whose scientific qualification is still yet to be decided.

The last kind of sentence qualifies the characteristic sentence of paraconsistent logic as pertaining to a theory in construction,

\footnote{Current formalist philosophy refrains the perception of paraconsistent logic; \( \) see for ex. Russell’s objections to an illustration of Vasiliev’s negation (G. Priest: "Vasil’év and Imaginary Logic", \emph{Hist. and Phil. of Logic}, \textbf{21} (2000) 135-146, p. 140); these objections want from Vasiliev’s negation what is a matter of clever generalizations, from a factual to a theoretical level; which instead formalists consider all at a same level.}
where the status of some theoretical sentences is provisional in nature.

Moreover, the above three kinds of sentences can be viewed as being three different versions of a principle belonging to an already completed scientific theory, when the sentence of this principle is considered from respectively the following three different viewpoints, which correspond to the three kinds of organisation of a scientific theory:

  i) an affirmative, idealistic axiom, as it occurs in an entirely deductive theory;

  ii) a methodological principle, as it occurs in a problem-based theory;

  iii) a scientific sentence without theoretical qualification, since the organisation of the theory is still yet undecided.

The history of scientific theories presents instances of these three versions of a scientific principle. The first version pertains to the most common organization of a scientific theory, the entirely deductive one; Zermelo's axiom and Newton's inertia principle\textsuperscript{11} constitute two instances of this kind of sentence. The second version pertains to previously listed theories, of a problem-based organization. The last case holds true when we refer to the pragmatic content, if any, of a principle without reference to its theoretical context; in other terms, when its theory is considered as a set of isolated sentences and formulas (in particular, when the inertia statement is considered on the Earth, where the centrifugal force is acting, although in a possibly negligible way).\textsuperscript{12}

An important instance of the third case is Lobachevsky's presentation of non-Euclidean geometry. By substituting "two straight lines meet" for Vasiliev's $A$ and "It is not true that two


\textsuperscript{12} J. Gidiemyn: "Geometric and Physical Conventionalism of H. Poincaré", \textit{St. Hist. Philos. Sci.}, 22 (1991) 1-22, illustrates Poincaré's view - and Adjukiewicz' one - according to which a scientific theory is under-determined by experimental data; hence, a sentence may be not entirely supported by experimental evidence.
straight lines do not meet" for \( \neg \neg A \), for Vasiliev's \( S \), the three Vasiliev's sentences describe respectively

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i) \quad \neg \neg A \rightarrow A, \text{ i.e. two hyperbolic secant lines,} \\
ii) \quad \neg \neg A \text{ fails to } \rightarrow A, \text{ i.e. two hyperbolic ultraparallel lines and} \\
iii) \quad \neg \neg A \rightarrow A \text{ and } \neg \neg A \text{ fails to } \rightarrow A, \text{ i.e. the two ultraparallel lines which meet at a point which is located at infinity, i.e. where all doubts are allowed. This last meaning is presented by Lobachevsky himself in his most relevant writing; he refers to the intersection of two lines at the infinite point by means of the following words: "In the uncertainty...,}^{13}
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just the meaning of Vasiliev's third kind of sentence. This interpretation of Lobachevsky's attitude vindicates Vasiliev's reiterated claim, i.e. his logic represents just the logic of Lobachevsky's geometrical theory.\(^{14}\) Moreover, it supports the above thesis; i.e. paraconsistent logic represents the subjectivist viewpoint of a scientist constructing a new systematic theory. Inside this process, a scientist operates by introducing a semantical divide between hard facts and heuristic principles. He then works at Vasiliev's "de re" level - through judgements about concepts (called "rules" by Vasiliev).\(^{15}\) According da Costa Puga's paper, that amounts to "handle two sorts of negations (logical and ontological)"; just as paraconsistent logic does.

One can add one more instance of this situation in Galilei:\(^{16}\)

"... but when the obstacle is removed,... I seem well that our mind is satisfied [= my mind guesses]... that impetus... would be so powerful to rise up the mobile to the same height. Hence, let us take provisionally that as a postulate, the absolute truth of which [= the attributing to it the role

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\(^{13}\) N.I. Lobachevsky: \textit{Geometrical}..., op. cit., prop. 16.


\(^{16}\) G. Galilei: \textit{Discorso su due nuove scienze}, Leiden, 1638, third journey (208) (emphasis added).
of an axiom] will be established by seeing that more results, constructed upon this hypothesis, well answer and compare with the experience."

A further, impressive instance is recognised in Einstein's paper founding special relativity. All basic problems of his theory are stated through five DNSs; moreover, he wrote the following well-known sentence: "Wir wollen diese Vermutung (deren Inhalt im folgende "Prinzip der Relativitaet" genannt werden wird) zur Voraussetzung erheben" (We will rise the conjecture (the substance of which will be hereafter called the "principle of relativity") to the state of a postulate). In a deductive theory this sentence constitutes an unsolvable puzzle. Actually, Einstein announces his promoting a Vasiliev's "indifferent" sentence (hence pertaining to paraconsistent logic) to the status of a methodological principle (a "postulate", i.e. a methodological principle inside a problem-based theory), and eventually to the status of an axiom-principle, all according to the above-mentioned, three kinds of organisation of a scientific theory.

In past times no one apperceived paraconsistent logic in this theoretical context since a clear distinction between an axiom-principle and a methodological principle was ignored. Hence, it was ignored the alternative kind of organisation of a scientific theory, i.e. the problem-based one. It was just the introduction of this distinction that allowed me to suggest the above interpretation of Vasiliev's paraconsistent logic.

5. Foundational context and Paraconsistent logic

In some past problem-based theories - e.g. in S. Carnot's thermodynamics - a *reductio ad absurdum* ends the list of DNS's.

It is commonly known that *ad absurdum* theorem does not belong to intuitionistic logic. Yet, in a PO theory a *reductio ad absurdum* on the thesis $T$ concludes no more than $\neg\neg T$, i.e. a sentence which denies the negation of $T$. It is rather a further logical passage, i.e. to equate $\neg\neg T$ to $T$, that violates intuitionistic logic, since in this logic $\neg\neg T \neq T$. Hence, when a *reductio ad absurdum* is deprived of this last passage, it does belong to intuitionistic logic.

Then, the conclusion $\neg\neg T$ has to be considered as a DNS representing a methodological principle for the following theoretical development. In other terms, the development of a problem-based theory, looking for merely methodological principles, does not need to equate the previous DNS, $\neg\neg T$, to a positive conclusion, $T$.

This kind of argument eventually gives, with respect to the theory at issue, an important result of the previous arguing inside non-classical logic. But at this point the author can choose: either to continue the theory in a heuristic way, but then contemporary scientists would dismiss this theory by considering it as a mere theoretical first effort, still lacking of certain results; or to change his important result as expressed by means of a DNS $\neg\neg T$, in a positive statement $T$; which he then considers as an *a priori* principle, from which to draw a deductive theory. What is implicit in this move, occurring after the *ad absurdum* arguing, is the change of organisation of the theory at issue, from a problem-based one to an entirely deductive one; and consequently the change of

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20 A. Markov (“On Constructive Mathematics”, *Trudy Math. Steklov*, 67 (1962) 8-14) suggested a principle performing the same passage, provided that it refers to a decidable existential predicate, obtained from an *ad absurdum* proof. In my opinion, Markov’s suggestion constitutes an attempt for attributing an universal framework to this passage through two clauses: i) to refer to an existential quantifier, a clause which qualifies the context as a theoretical one; ii) to be consequence of *ad absurdum* proof, a clause which qualifies the organization as a problem-based one. Instead, the decidability of the predicate does not qualifies the context as a scientific one, and rather it appears as a misleading clause.
the nature of the principle at issue, from a methodological principle to an axiom-principle; at last, this move constitutes a change of logic too, from the non-classical one to classical logic.

Lobachevsky did this move just through the final words of proposition 22 of his booklet about the two possibilities on the parallelism angle; all subsequent propositions are drawn from the axiom-principle of a parallelism angle lesser than $\pi/2$. The same did S. Carnot after his celebrated *ad absurdum* theorem; there he started a theoretical development where the efficiency of a heat engine was *a priori* considered as sensibly lesser than 1.21

When we saw this move from the outside of the whole theory, it appears as a mixing two kinds of organisations; in a more rigorous way, as a theoretical co-existence of two different kinds of principle, methodological principles and axiom-principles. At last, the move constitutes a juxtaposition of two different kinds of logic - more precisely, two kinds of negations at the same time. That means a paraconsistent logic.

I think useful to introduce this seemingly use of paraconsistent logic in order to emphasise how much in the past it was difficult to scrutinise the foundations of a scientific theory and moreover in which subtle way paraconsistent logic was apperceived, in an obscure way, by scientists.

An effective use of arguing according to paraconsistent logic can be recognised again in S. Carnot's book. His theory followed the incorrect caloric theory (according to which heat is a state function, i.e. $Q=0$ in a cycle). However, in a footnote in p. 37 S. Carnot declared to be dubious about this hypothesis, just Vasiliev's third kind of sentence. However, this doubt did not refrain S. Carnot from discovering the basic laws of the new theory, through a clever use of the two levels of negation.22

Even subsequent thermodynamic theory, as completed by Clausius and Kelvin in 1850, includes an use of paraconsistent logic.

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logic. Its first axiom-principle states $L = Q$ (when $DU=0$); then its second principle contradicts this equality, since not all $Q$ can be converted in $L$; in other terms, between $L$ and $Q$ the symbol of equality $=$ represents an exchange in the direction from $L$ into $Q$ only, not vice versa. Rather the second principle may be correctly stated as follows: $\neg (L \neq Q)$ (in words: "It is not true that heat is not work"); notice that this sentence represents the corresponding methodological principle of the above version of the first principle of thermodynamics. Thus, the two versions of the first principle, taken together, verify third Vasiliev's sentence. Instead, by a sophisticated way of formalising these principles by means of both clever words and mathematical terms a textbook presents modern thermodynamics as a seemingly deductive theory in classical logic.

Norton stressed that in 1900 paraconsistent logic was effective in the construction of black-body theory. In fact, no one physicist opposed this theory in the name of classical logic. Apparently, in order to be successful, all physicists allowed provisional logical inconsistencies.

Paraconsistent logic occurs in the subsequent quantum theory too. It is well-known that the logic of Quantum mechanics is not a classical one. The proof of this fact is obtained by contrasting the logic of Hilbert subspaces, clearly a classical one, with an implication from the indeterminacy principle; which, being an inequality, cannot belong to classical logic preserving equality. Actually, in the usual formulation of Quantum mechanics both Hilbert space and indeterminacy principle co-exist. This fact is the ultimate reason for von Neumann paradox. Which is commonly ignored, by giving reality to Hilbert space only; whereas the

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23 Several authors expressed perplexities on the current versions of these two principles. I mention the easiest criticism: M. Jona: "What is Energy?", Physics Teacher, 22 (1984) p. 6.
indeterminacy principle is relinquished to an final requirement of well-adequacy of the theoretical calculations to reality.

6. Paraconsistent logic in the proof of first Goedel's Theorem

Let us consider now a case of effective use of paraconsistent logic in a crucial point of the most hard science, i.e. Goedel's original proof (=GP) in mathematical logic.²⁶

I start by listing some basic facts:

1) According to Hilbert program, the final proof of the consistency of arithmetic had to be performed by joining a mathematical theory, i.e. Peano's formal system P, with a metamathematical theory (= MM).

2) Unfortunately, MM was never illustrated in a satisfactory way by Hilbert. However, he claimed that its kind of mathematics is of a "finitist" nature.²⁷ Goedel interpreted this finitist mathematics as the theory of (primitive) recursive functions.

3) Moreover, Hilbert recognised that in MM the LEM cannot be applied, hence logic is an "intuitive, informal logic". In GP the goedelisation process codifies sentences belonging to P as well as some other elements of MM, into integer numbers.

4) GP numeralwise expresses 46 relationships, among which 45 are proved to be recursive. They include classical propositional logic (no.s 13, 14, 32; the fourth relationship of no. 32 is the classical double negation law), the principle of complete induction (no. 34) and even the comprehension axiom (no. 40). Finally, a non-recursive relationship, no. 46, expressing the crucial predicate "to be provable", is added.

5) GP includes in an essential way the notion of ω-consistency. This notion has no room in classical logic, since to write "for each value n : P(n)…" is exactly the same than to write "∀nP(n)". A difference between the above two expressions is instead possible when our logic is not a classical one, e.g. the intuitionistic one.

From 3) one has to conclude that the logic in MM is at most intuitionistic logic; after 70 years of research upon the foundations of logic, no different logic could be a candidate for playing such a role. From both 3) and 6) we have to conclude that in GP two kinds of logic are involved at the same time, the classical one and the intuitionistic logic. Evidence comes from Kleene's introduction when he is presenting the definition of "numeralwise expressible" by the following formula \( \neg R(x) \rightarrow \{ \neg R(x) \text{ is provable in } P \} \), where two kinds of logic are joined in a single formula, i.e. the logic of

the informal negation and the logic of "formal", classical negation \(\neg\); actually, in this formula the symbol of implication transcends both kinds of logic.

We have to conclude that GP deals with two kinds of negations at the same time, a "de re" (ontological) negation belonging to MM, and a "de dicto" (logical) negation in \(P\). We know that this fact characterises paraconsistent logic, as it was formalised by Puga and da Costa.\(^{29}\)

Is Goedel's conclusion in agreement to paraconsistent logic? Surprisingly, the answer is yes. When we accept to be in a paraconsistent logic, i.e. to include inside the theory, governed by classical logic, the non-classical logic of MM, then from the notion of \(\omega\)-consistency we obtain a contradiction. Indeed, in intuitionistic logic, i.e. the logic of MM, the two predicates \(\forall n \neg P(n)\) and \(\neg \exists n P(n)\) are not equivalent and the definition of \(\omega\)-consistency, usually concerning the former version, cannot be reiterated by means of the latter version. Instead, in classical logic a predicate \(\forall n \neg P(n)\) is equivalent to \(\neg \exists n P(n)\). As a consequence, the whole theory, by including a sentence (equivalence) and its formal negation (not equivalence), is inconsistent (when we are reading the two sentences in formal terms). When on the contrary we do not recognise to be in a paraconsistent situation, we proceed in GP by using of classical negation only, except in the notion of \(\omega\)-consistency; then the subsequent part of the original version of GP proves that the theory \(P\) is incomplete, being \(P\) unable to include what is meant by co-consistency.\(^{30}\)

As a general conclusion, GP is falsely a proof inside the original Hilbert's program where MM was lacking. Rather, by

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\(^{29}\) Under this light the goedelisation process wants to merge the "de re" negation inside \(P\), i.e. to identify it with the "de dicto" negation: a just subsequent Goedel's paper proved that this immersion is impossible. K. Goedel: 1932.

introducing MM it constitutes an effort in order to argue inside paraconsistent logic.

In the previous instances of paraconsistent arguing inside the foundational context, I showed that this kind of logic is possible when we are comparing two mutually incommensurable theories with respect to their kinds of organization; in the present case of GP, the two theories are the two kinds of arithmetic, both represented by the same sentence, i.e. the notion of $\omega$-consistency, as considered according to the two meanings of negation. Hence, GP case is more interesting than previous ones, since the paraconsistent arguing applies to a subject which is not merely a principle belonging to a single theory, but to a whole mathematical theory (arithmetic), i.e. to a subject of a formal nature.

One has to test whether all kinds of proofs of Goedel's thesis essentially belong to paraconsistent logic. Only an affirmative answer to this enquire will allow to conclude that GP essentially belong to paraconsistent logic. In this case one would reach a new appraisal of GT, as appealed by da Costa and Doria.31

Conclusions

20th Century was the century in which scientists apperceived the relevance of paraconsistent logic, in a first, direct way by Vasiliev and few others. At the same time the occurrences of paraconsistent logic in black-body theory, quantum mechanics and GP suggested, in a very obscure way, the same kind of logic to scientists' community. This obscurity was attributed to paraconsistent logic itself; rather it was not made clear in which context paraconsistency holds true.

Most scholars then studied paraconsistent logic under the aspect of its introducing contradictions; that means the failure of

the law of excluded middle. Rather, in this paper the interpretation according to the failure of the double negation law resulted more productive and more adequate to the original tradition of this kind of logic.

In the above I argued that an incommensurability phenomenon between two theories leads paraconsistent logic. In fact, not before 20th century it was manifest that two theories - e.g. classical mechanics and quantum mechanics - could be mutually incompatible in a radical way. Actually, this incommensurability phenomenon can be traced back to a century before, to the birth of non-Euclidean geometry in Mathematics; whereas in theoretical physics to the birth of thermodynamics, which is incommensurable with Newtonian mechanics. Hence, the consistent use of paraconsistent logic can be traced back to the first decades of 19th Century, as Vasiliev imagined it.

From all the above I conclude that paraconsistent logic constitutes an essential tool for both a scientist and a historian of science inasmuch as it allows to evidentiate a series of problems which otherwise would result as invisible.