On Existential Quantifier

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I. In English logical language the expression

«”There is” …»

is commonly used to denote what an Italian logician would denote in terms of

«“It exits”…»

Both, and similar other expressions, lead to the following problem: if

i) "∃x"

is read as

i.i) «There is at least one x …»

and if i.i) is meant to denote

i.i.i) «It exists at least one x …»

what kind of existence we are referring to?

This paper deals with said problem; needless to add, the kind of existence I am going to refer is not the ontological but merely the logical one. I mean that when I refer to an object (in the widest sense of the term) as “existent” I intend nothing more ontologically committed than this: said object is part of the Universe of Logical Discourse (U.L.D.). I am not going to consider any other way an object may exist; this in order not to by-pass the border of logic and ontology.
To identify existence ad “being part of U.L.D.” makes a sentence like

1) «“a” exists»

semantically equivalent to

1.1) «“a” denotes an object [which is part] of U.L.D.»

An acute solution for the problem of the logical meaning of the term “exists” can be found in *Languages of Possibility* by Graeme Forbes;¹ I say “acute” because his interpretation of “∃…” shows a way to make use of possible worlds semantics which is, as far as I know, unprecedented. As it will be better seen in the next pages, by coupling said semantics with Aristotle's concept of possibility Forbes succeeds in finding a very good solution for the problem of how introduce, without unwanted logical consequences, fictional object in U.L.D.

As far as the content of this paper is concerned, the next two paragraphs will deal with what Forbes calls “the possibilist interpretation” of

i.i.i) «It exists …»

and to what consequences can be derived from said interpretation, while the forth paragraph will deal with some objections that may be risen when widening the domain of individual variables to include fictional beings. I am not going to consider, here, the so-called “substitutional interpretation of quantifiers” because, as it will better seen in the *locus ad hoc* of this writing, it appears to me as a way to avoid, rather than solve, said problem.

II. Quite a few pages of Forbes 1989 are aimed at criticizing what he calls “the actualist interpretation of existential quantifier”,

namely, the point of view (chiefly defended by Willard v. O. Quine)\(^2\) that the quantifier in expressions like

\[ \varphi = \text{a function whatever} \]

\[ (\exists x) \varphi x \]

should be taken as denotative of the actual existence of at least one individual such as that individual may satisfy the function denoted by

\[ \varphi \] (in which case 1) is true)

or else (aut) may not satisfy said function (which makes 2) false). Said interpretation originates from the problems posed by the relation (which corresponds to a logical law)

\[ \sqrt{\varphi. qa} \rightarrow (\exists x ) q x \]

when “a” denotes a fictional object, that is, an object [of U.L.D.] which does not really exist or, as I prefer to say, “which does not occupy an independent position in a space-time contest”.

Let us assume, for instance, that “a” stands for “Santa Claus”; in such a case 3) appears as [implicitly] stating that from the fictional (non real) existence of “Santa Claus” can be deduced that there is at least one object [of U.L.D.] which does really (non fictionally) exist. As we are going to see in the next pages, all other problems concerning the introduction of fictional objects in U.L.D. seem to take origin from the apparent incongruity of 3).

In the second half of the XIX century the founder of modern quantified logic, Gottlob Frege, afforded the problem if fictional objects should or not be introduced in U.L.D. and solved it in a negative way. In Frege’s opinion sentences concerning fictional objects do not express propositions because they can not denote

neither truth nor falsity; consequently, said sentences are void of logical interest. More specifically; according to Frege, proper names (be they grammatical or logical) can be symbolised by individual constants (and, in this way, introduced in U.L.D.) if and only if they denote existing objects because in this case (and this case only) a proper name has not only a sense (Sinn) but also a meaning (Bedeutung; denotation). Proper names denoting fictional object do have a sense but are void of meaning; consequently, sentences concerning fictional objects cannot be qualified as true/false and must, for this reason, be considered void of logical interest.

In my opinion, quite a few non Fregean logicians agree with him on the aforementioned point; this, I presume, in order to avoid all problems which the introduction of fictional objects in U.L.D. may cause. Surprisingly enough, this is not the case for Forbes. I say “surprisingly” because Forbes is a Fregean logician, yet he considers the introduction of fictional objects in U.L.D. not only possible but also advisable.

In fact, Forbes agrees with Frege on quite a few, basic logical points like, for instance, the identification of an object-proposition as a logical entity denoting “Truth” and “Falsity”, in defending the law of bivalence on the base of the assumption that “true” and “false” are logical names of any proposition and on quite a few more points of Frege’s logic which are not mentioned here because not connected with the topic of this paper. Yet, as already seen in par. 1), Forbes is very critical when it comes to the actualist reading of

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4 Like definite description or any other expression apt to denote one, and only one, individual.
6 I mean, logicians which disagree, if non completely, at least mostly with Frege’s logical points of view.
7 See, in part., Forbes 1989, pp. V-VIII; in introducing the content of said volume, he states that the «….. approach developed is broadly Fregean».
8 Compare Frege (Mangione) with Forbes 1989, pp. 19-44.
i) \( (\exists x) \ldots \),
namely, when it comes to reading i) as denoting the actual existence of at least one individual such as …… On this point, Forbes’ criticism appears to be directed towards Quine rather than Frege; I suppose this is due to the fact that, unlike Frege, Quine considers void of logical interest not only sentences concerning fictional (non real) objects but also all sentences concerning abstract entities\(^9\) (and modalities, too).\(^10\)

According to Forbes, actualist interpretation of i) is too narrow for it compels to ban from U.L.D. not only objects (like, e.g., “the round square” “my enemy friend”) which cannot possibly exist for their concept is self-contradictory, but also objects which do exist, though only in a fictional world.\(^11\) It is true that fictional objects do not exist in the same way actual (non fictional) objects do; yet they are brought into existence by the creators of the stories they first appear in and cease to exist only when the memory of such stories is lost.\(^12\) Let us consider, for instance, the fictional object denoted by the name “Snow white”. Said name does not denote any actually existing being; yet “Snow white” was brought into existence by Charles Perrault, who devised the fairly tale bearing that name, started to [objectively] exist the moment said tale was first made known by its creator and will cease to exist only when the name "Snow white" will appear as meaningless as, e.g. “the firblifagus”, (that is, only when memory of said tale will be lost). Forbes recalls the reader attention to the fact that, to laymen, statements concerning fictional objects (or, if the reader so prefers, “exhibiting the name of a fictional object in a subjective position”) are commonly perceived as being true or false according if they do

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\(^9\) See, for instance, Quine 1953, pp. 198-203.
\(^10\) See, for instance, Quine 1966/1977, pp.120-128 and compare with Forbes 1989, pp.78-114
\(^12\) See Forbes 1989, p. 23.
or do not express this or that quality possessed by said objects in the fictional world they are part of. Let us consider, for instance

4) «Othello is a jealous husband»
5) «Desdemona is an unfaithful wife»

Should we ask a logician to say what truth-value can be given to 4) and 5), he/she would certainly start to answer by raising the question of what kind of existence can be ascribed to “Othello” and “Desdemona” (or, to be exact, to the objects denoted by said names): this because the logician’s answer depends primarily on the way existence is conceived. Yet if we pose the same question to a layman then, provided he knows that “Othello” and “Desdemona” denote two characters of Shakespeare’s “Othello” an does not totally ignore what that play deals with, that layman will not hesitate to qualify 4) as “true” and 5) as “false”. Besides, if asked to explain why, his answer will, more or less, amount to that: because Shakespeare depicted Othello as a jealous husband but depicted Desdemona as a faithful wife. In other words, 4) is judged “true” because, in the fictional world created by Shakespeare (world corresponding, in its totality, to the play bearing the name of his main character) Othello is depicted as a jealous husband and 5) is judged “false” because, in said world, Desdemona is depicted as a faithful wife. Needless to add, both (and similar ones) ways to justify the assumption that 4) is true and 5) is false originate from the following logic principle: for any given sentence of the kind

«$\varphi a$»

said sentence is true if the argument “$a$” satisfy the function “$\varphi$”, false otherwise.

If it is the case of what previously said, then we may agree with Forbes when he states that common language is far from being void of statements concerning fictional entities and judged as true

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or false. Besides, adds Forbes, laymen do not perceive as self-contradictory but, rather, as true all statements (like, for instance «Hercules does not exist») denying the existence of some (at least one) fictional object and they perceive not as [logically] undecidable (neither true nor false) but, rather, as false all statements (like, for instance «Pegasus exists») asserting the existence of one or more fictional objects. In Forbes’ opinion, this happens because laymen perceive said statement not as denying/asserting existence in toto but as denying/asserting a particular, specific kind of existence: the real (non fictional) one.\footnote{See Forbes 1989, pp. 22-23.}

From his investigations on common language Forbes deduces the following conclusions: \(a)\) the term “existence” is immediately perceived as denoting an undifferentiated way of being; \(b)\) if it is so, then that of existence corresponds to a primitive concept, while real existence and fictional existence correspond to secondary (non primitive) concepts because \(c)\) the distinction from real and fictional objects takes place after reflection on said primitive concepts; in other words, said distinction is subsequent.

The primitive character of the concept of existence, coupled with possible worlds semantics and the Aristotelian definition of possibility as “what is or might be”, lead Forbes to identify logical existence as “being part of some possible world” and to introduce

\[
\text{ii)} (\Sigma x) \ldots .
\]

as a mean to denote

\(\text{ii.i)} \text{ «In some possible word there is at least one } x \text{ such as…»}

In my opinion, \(\text{ii)}\) can also be read in terms of

«There possibly is at least one \(x\) such as …»

or in the semantically equivalent terms of
«There is at least one possible $x$ such as …»

Needless to add, none of said readings requests the addition of the operator of possibility; as

«$\Sigma$…»

is, by definition, denotative of possible existence, to add the aforementioned modal operator to any sentence containing

ii) $(\Sigma x)$

would be redundant.

The domain of possible existence may include, in my opinion, other ways of existence besides real and fictional one; as will be seen in the next paragraph, all fictional beings may be associated to the domain of mental entities but not vice-versa. Yet, the only kind of existence Forbes deals with is the real and fictional one; this makes ii) semantically equivalent to

ii.i) «There is at least one real or fictional $x$ such as …»

According to Forbes, the introduction of ii) and its reading in terms of ii.i) offers an easy and congruent way to solve evaluation problems posed by sentences concerning fictional object and, for this reason, to solve problems arising from the introduction of said object in U.L.D. The best known of said problems originates when the subject of a statement is a fictional object and the predicate denotes a non actually existing quality, as it happens in

6) «Pegasus is a flying horse»

If existence is assumed as actual (non fictional) the translation of 5) in terms of
“$H_f$” = “is a horseflyer”\textsuperscript{15}
“$\rightarrow$” = “implies”

6.1) « $H_f a$ »

leads to

7) $\neg f, H_f a \rightarrow (\exists x)H_f x,$

that is, to a sentence implicitly asserting that from the non actual existence of a given object [of U.L.D.] possessing an equally non actual quality it can be deduced the actual existence of at least one object possessing said quality. Nor translation of 5) in terms of

“$F$” = “is a flyer”
“$H$” = “is a horse”
“$\wedge$” = “and”

6.2) $F a \wedge H a$

seems to help solve problems posed by the introduction of “Pegasus” in U.L.D., for, from a semantic point of view, 6.2) appears as ascribing to a fictional object two qualities (“being a flyer”, “being a horse”) which, separately considered, do actually exist “… with what follows …” when it comes t

6.3) $F a \wedge H a \rightarrow (\exists x)F x \wedge A x$

Similar problems of self-congruence arise when a) the subject of a sentence is a fictional object while the predicate denotes a quality which can be either real or fictional and b) the subject is an object which can be either fictional or real. Let’s consider, for instance,

\textsuperscript{15}For the notation here adopted for complex predicates see Stahl 1981, pp.303-305. Following Quine 1984 I call “complex” predicates semantically equivalent to the logical products of two or more simple predicates; I call “simple” any predicate which can not be analysed in said terms.
7) «Jago is a traitor»

Here the predicate “is a traitor” is not univocally denotative for it may denote a quality which can be either the quality of a real being or the quality of a fictional being; in the former case, said quality is real, in the latter, fictional. Now, the name

\[ “T” = “\text{T}raits” \]
\[ “\hat{x}” = “\text{The} x’s \text{such as …}” \]

\[ \hat{x}.Tx \]

denotes a non empty class both in the case of fictional or real treason; consequently, translation of 6) in terms of

\[ “T” = “is a traitor” \]

7.1) \( Ta \)

leads to

\[ \forall \, Ta \rightarrow (\exists x)Tx \]

In other words, the introduction of 6) in U.L.D. seems to carry with it the following consequence: from a fictional quality ascribed to a fictional object can be deduced the existence of a real quality to be ascribed to at least one real object. Let us consider, again

8) «Vergil acts friendly with Dante»

In 8), “Vergil” and “Dante”, though being proper names, are not univocally denotative for they may denote, respectively, not only the names of the real Latin and the real Italian poet (which makes said sentence false) but also the names of two characters of Dante’s “Divine Comedy”, in which case 7) is true. Consequently,
the introduction of 7) in U.L.D. seems to bring with it the following, unwanted consequence: the same sentence may be qualified as “true” and “false” (“... with what follows” when in it comes to the law of non-contradiction). Needless to add, logical problems posed by 7), 8) and similar sentences can be solved by making explicit reference to the pragmatic dimension, that is, to the emission contest of said sentences. In the case of 6), logic problems arising from the introduction of said sentences in U.L.D. vanish if it is replaced by

7.1) «One of the characters of Shakespeare’s “Othello”, named “Jago”, is depicted as being a traitor» which makes explicit that the treachery attributed to the subject of 7) is as fictional as said subject itself. As far as 8) is concerned, reference to the contest of emission leads to replace it with

8.1) «One of the characters of Dante’s poem “Divine Comedy”, denoted by “Vergil”, befriends (=acts friendly with) another character of said poem, named “Dante”»

which is true, or with

8.2) «The Latin poet denoted by “Vergil” befriends the Italian poet denoted by “Dante”»

which is false.

As 8.1) and 8.2) are two different statements, there is no denial of the law of non-contradiction in qualifying 8.1) as “true” and 8.2) as “false”; in fact, what we qualify as “true/false” is not the same sentence but, rather, two different tokens of a sentence. Yet, as I am going to show in the next paragraph, possibilist interpretation of existence offers a more economical way to solve logic problems posed by 7) and 8); I say “more economic” for a) it makes needless any explicit reference to emission contest, thus
making translation easier and b) it allows, but does not compel, to assume that what is qualified as “true/false” are sentence-tokens rather than sentences or, better, statements.

III. According to Forbes, any problem arising from the introduction of fictional objects in U.L.D. originates not from said introduction in itself, but from the marked ontological commitment implied by the actualist interpretation of the existential quantifier. Possibilist interpretation of said quantifier and, consequently, the use of

ii) $\Sigma x$

bears a minimal ontological commitment because ii) denotes an existence which may be either actual [real] or fictional [possible]. If it is so, than any statement of the form

$$(\Sigma x)q x$$

bears no ontological commitment more marked than assuming that the domain of the variable “$x$” does not include objects whose concept is self contradictory; that is, objects (like “the enemy friend”, “the round square”) whose existence is absurd, impossible.

Let us see, now, if we may or may not agree with Forbes when he says that possibilist interpretation of existential quantifier allows to introduce harmlessly (that is, without any unwanted logical consequences) fictional beings in U.L.D., thus enriching the domain of logic. According to Forbes, said interpretation proceeds from (and is, so, justified by) a possibilist concept of existence, that is, from an identification of existence as “being part of a possible word”; according to said way to conceive existence, to say that a certain object “$a$” exists is semantically equivalent to say that said object is part of a possible word. According to Aristotelian definition of what is possible (what is or else may be) to affirm that an object “exists” comports no greater ontological commitment than to assume that the concept of said object is not self-contradictory and, for this reason, [logically] impossible.
Let us consider again the statement

5) “Pegasus is a flying horse”

and let us translate it in terms of

5.1) $H_f a$.

Possibilist interpretation of existence does not, in itself, allow to ascribe to “$a$” any kind of existence than that implied by the expression “being part of a possible word”. Consequently, should

i) “$(\exists x)$”

denote actual existence, from 5.1) can not correctly be deduced neither

6.1) $H_f a \rightarrow (\exists x) H_f x$

nor

6.2) $H a \land F a \rightarrow (\exists x) H x \land F x$;

what can be correctly deduced by 5.1) is

9) $H_f a \rightarrow (\Sigma x) H_f x$

(“If “$a$” is a flying horse, then there possibly is something which is a flying horse”) or is

9.1) $H a \land F a \rightarrow (\Sigma x) H x \land F x$

(“If “$a$” is a horse and a flyer, then there possibly is something which is a horse and a flyer”)

So, should 9) and 9.1) be asserted, this would imply no ontological commitment more marked than that: if a given possible object of U.L.D. can be ascribed some (at least one) given
quality, the from it can be deduced the possible existence of at least one object of U.L.D. possessing said quality.

As far as statements in 7) and 8) are concerned, to state

7.1) «Ta»
8.1) «Bab»
“B” = “befriends” [=”acts friendly”]

implies no more marked ontological commitment that to assume that “a” in 7.1) and “a” and “b” in 8.1) are, as Forbes says, “part of a possible word” (that is, objects which do actually or fictionally exist). Consequently, from 7.1) can correctly be deduced only

\((\Sigma x)Tx\)

that is, a statement asserting that there is at least one possible individual to which can be ascribed the quality of being a traitor. Again, from 8.1) can correctly be deduced only

\((\Sigma x)(\Sigma y)Bxy\)

that is, a statement asserting that there possibly are at least two individuals such as the first befriends the second.

The reader might object that, as

i) \((\Sigma x)\)

denotes an undifferentiated kind of existence, this may cause confusion between objects of U.L.D. whose existence is as fictional as the qualities ascribed to them and objects of U.L.D. whose existence is real. According to Forbes, actual (non fictional) existence may be denoted, when needed, by using a different notation; his suggestion is to use

i) \((\exists x)\)
to denote actual existence\(^\text{16}\). As far as the universal quantifier is concerned, Forbes suggest to use

\(\Pi x\)

to denote «For all possible “x’s”»

and to use

\(\forall x\)

to denote «For all actual “x’s”»

I would suggest to use a notation \textit{ad hoc} to denote, when needed, fictional existence only; I’m not hinting that it is necessary, only that it is more economic.

Let us consider, for instance

9) «If Jago is a traitor, then something is a traitor»

and let us assume that the speaker of 9) wants to make clear that he is talking of a fictional quality to be ascribed to a fictional object.

Translation of 9) in terms of \(\exists\)

9.1) \(T a \rightarrow (\exists x)T x\)

does not help because what is denoted by

\(\Sigma x\)

is an undifferentiated (actual or fictional) way of existence, which makes equally undifferentiated the existence of the quality denoted by “\(T\)”. Following Forbes’ suggestion, 9) could be translated in terms of

\(^{16}\) See Forbes 1989, pp. 33.
9.2) \( Ta \rightarrow (\Sigma x)Tx \land \sim (\exists x)Tx \)

[“\( a \)” is a “\( T \)” implies there is at least one “\( x \)” such as “\( x \)” is a “\( T \)” and there is no actually existing “\( x \)” such as “\( x \)” is a “\( T \)”]

that is, in terms of an expression explicitly denying that “\( x \)” varies on individuals which do actually exist; as possible existence is conceived, by Forbes, as fictional or else actual, to affirm 9.2) is semantically equivalent to state that the \( x \)’s which may satisfy “\( T \)” are fictional objects. Needless to add, if the consequent of 9.2) concerns at least one fictional object, so should the antecedent.

I think, anyhow, that in cases similar to the one in 9) it is better to make explicit reference to the kind of existence implied. This may be done in two ways: a) by making use of a predicate “\( ad \ hoc \)” or b) by making use of quantifiers \( ad \ hoc \). By making use of the notation here adopted for complex predicates, 9) could be translated in terms of

“\( E_f \)” = “exists fictional”

9.3) \( Ta \rightarrow (\Sigma x)Tx \land E_f a \)

(“\( a \)” is a “\( T \)” and there is at least one possible “\( x \)” such as “\( x \)” is a “\( T \)” and “\( a \)” exists fictionally) that is, in terms of a sentence explicitly affirmative of the fictional existence of “\( a \)” and implicitly affirmative of the fictional character of “\( T \)”.

By extending to quantifiers the notation here adopted for complex predicates, 9) can also be translated in terms of

“\( \exists_f \)” = “there is at least one fictional”

9.4) \( Ta \rightarrow (\exists_f)Tx \)

a translation, this one, which makes evident the fictional character of the quality denoted by “\( T \)” and makes implicit clear that “\( a \)”
denotes a fictional object. Needless to add, the choose between translation modalities in a) and b) depends on contest.

Let us consider, again,

8.1) \( \text{Bab} \)

This statement contains no quantifier, which, in my opinion, allows to translate it as follows

8.2) \( \text{Bab} \land E_f a \land E_f b \)

(that is, in terms of a sentence explicitly affirming that “a” and “b” denote a couple of fictional objects)

or in terms of

“\( E_a \)” = “exists actually”

8.3) \( \text{Bab} \land E_a a \land E_a b \)

that is, in terms of a sentence explicitly affirmative of the actual existence of “a” and “b”. Needless to add, 8.2) and 8.3) are not the same sentence translated in two different ways but, rather, two different statements denoting two different interpretations of 8).

Let, now consider,

10) \( \text{Bab} \rightarrow (\Sigma x)(\Sigma y)Bxy \)

and let us assume that we want to make explicit the twin sense (\( \text{Sinn} \)) which can be ascribed to that sentence. This goal may be reached by translating 10) in terms of

11) \( \text{Bab} \rightarrow (\Sigma x)(\Sigma y)Bxy \land E_x \land E_y \)
12) \( \text{Bab} \rightarrow (\Sigma x)(\Sigma y)Bxy \land E_a x \land E_a y \)

or in the simpler terms of

11.1) \( \text{Bab} \rightarrow (\exists x)(\exists y)Bxy \)
11.2) $Bab \rightarrow (\exists x)(\exists y)Bxy$

In Forbes’ notation, “$(\exists \ldots)$” denotes actual existence; I prefer to adopt it to denote simple (undifferentiated) existence and to use “$(\exists_f \ldots)$” and “$(\exists_a \ldots)$” to denote, respectively, fictional and actual existence.

In my notation 10), 11.1) and 11.2) would be expressed, respectively, in terms of

$$Bab \rightarrow (\exists x)(\exists y)Bxy$$
$$Bab \rightarrow (\exists x)(\exists y)Bxy$$
$$Bab \rightarrow (\exists x)(\exists y)Bxy$$

Let us consider, now, the sentence

13) «Wiches do not exist»

I suppose the reader will agree with me that 13) can not be correctly translated in terms of

"W" = “is a wich”

$$~(\Sigma x) \ Wx$$

or in the equivalent terms of “~" = “...not…”

$$(\Pi x)\overline{Wx}$$

because what is meant to be denied is not existence in toto but the actual existence of any object-wich.

Possible existence is conceived, by Forbes, as a mutually and exhaustive alternative between actual and fictional existence; consequently, 13) can be correctly translated in terms of

13.1) $~(\exists x) \ Wx$
or in terms of

13.2) \((\forall x)\neg Wx\)

or in any of the alternative ways suggested in this paragraph.

IV. To sum up the content of par. 2) and 3), I think Forbes is right in assuming that logic problems arising from the introduction of fictional objects in U.L.D. depend on assuming an expression like

“(∃…)”

as denotative of

“there actually is ….”

I hope the reader will agree with me that possibilist interpretation of the existential quantifier helps avoid said problems, thus causing an enrichment of the domain of logic evaluation. Needless to add, Forbes possibilist interpretation of existence is, as far as I know, the most recent but not the only way to introduce fictional objects in U.L.D. *salva congruitate*.

In par. 1) I said I would not consider, here, the so-called “substitutional interpretation” of quantifiers because it appears to me as a way to avoid, rather than solve, problems connected to the aforementioned introduction. According to said interpretation, expressions like

\((\exists x)\phi x\)  
\((x)\phi x\)

should be read in terms, respectively, of
«(for at least one “x”) “x” satisfies “ϕ”»
«(for every “x”) “x” satisfies “ϕ”»

thus avoiding any reference to existence, or, more exactly, to the kind of existence that can be ascribed to the objects contained in the domain of the variable “x”. As we have seen in the last paragraph, reference to the kind of existence (and, for this reason, to existence) to be ascribed to said objects can not always be avoided; when this happens, reference to existence, “thrown away from the door” of existential reading of quantifiers must be “reintroduced from the window” of a predicate ad hoc. I mean a predicate like “E_f” (“exists fictionally”) or “E_a” [“exists actually”]. Forbes’ introduction of “Σ…” to denote possible existence appears more economical to me for it allows, but does not compel, to denote the kind of existence ascribed to the objects which belong to the domain of an individual variable by mean of a complex predicate. I mean that, for any statement of the kind

\[ (\Sigma x) \phi x \land E_a x \]

\[ (x) \phi x \land E_f x \]

“E_a …” and “E_f …” correspond to a sort of shorthand notation to denote, respectively,

“\textbf{A}” “is actual”
“\textbf{F}” “is fictional”

i)«….. \textbf{A} (\exists x)…..»
As “$E_a \ldots$” and “$E_f \ldots$” are functions which can be satisfied only by all individuals which can satisfy the functional expression “$A (E\ldots)$” and “$F (E\ldots)$”, (and vice-versa, of course) we have

16) \[ \sqrt{f}: (x) E_a x \equiv A (Ex) \]
17) \[ \sqrt{f}: (x) E_f x \equiv F (Ex) \]

Equivalences in 16) and 17) allow to substitute, salva veritate, expressions containing second order functions with expressions containing complex functions\(^{18}\), thus making translation in symbolised language simpler. Of course, while they are commonly adopted, complex function can not (and in fact, are not) introduced in symbolised language in all cases because they do not cover the whole field of deduction covered by translation by mean of complex functions. Let us consider, for instance, a sentence like

18) «If someone exists actually then someone exists»

18) expresses, in common language, a logical law; but this law can be introduced in logic language only if 18) is translated in terms of

18.1) \[ \sqrt{f}: (\exists x) A (Ex) \rightarrow Ex \]

As already shown in par 3), what can be expressed by mean of the complex predicates “$E_a \ldots$” and “$E_f \ldots$” can be salva veritate expressed by mean of the complex quantifiers “$(\exists_a \ldots)$”

\(^{17}\) For the notation here used to denote second order functions, see Copy 1979, pp.154-155.

\(^{18}\) On complex functions see, in part, Quine.
and “(∃f….)”. Needless to add, translation by mean of complex quantifier can not be adopted when deduction procedures compel to make explicit reference to second order functions; besides, this kind of translation can not be adopted when the argument of a function is an individual constant. Yet, in the case of sentences like those in 14), 15) and in all similar cases, I am of the opinion that translation in terms of

19) (∃a)xφx
20) (∃f)xφx

is simpler and, for this reason, more economic than translation in terms of 14) and 15).

Another way to introduce fictional objects, salva congruitate, in U.L.D. is due to Leonard and consists in assuming as “necessary” all qualities ascribed to fictional object and as “possible” (“necessary, or else, contingent”) all qualities of real objects19. I consider this sound because, as fictional objects are creation of the human mind, they could have been ascribed qualities different from the ones they are said to posses only as long as they existed just in the mind of their creators; once they are brought to objective existence (that is, once they become part of a poem, a play and so on) they can not possibly acquire qualities different from the ones their creators ascribed to them (or loose one of said qualities). If this is accepted, than a sentence like

\( Fa \rightarrow (Ex)Fx \)

would denote “a” as a fictional, or else real (non fictional) object of U.L.D., while sentences like

“□” = “necessarily”
“◇” = “possibly”

\( \square F_a \rightarrow (\exists x) \square F_x \)
\( \Diamond F_a \rightarrow (\exists x) \Diamond F_x \)

would denote, respectively, that “\( a \)” is a fictional/a real object.

I suppose the reader will agree with me in judging Forbes’ solution to the problem of how introduce fictional objects in U.L.D. salva congruitate is more economical than Leonard’s. I say so because while Leonard’s solution compels to evaluate sentences concerning fictional objects by mean of modal logic, Forbes’ solution does not.20. Last but not least, Leonard’s way to introduce fictional object in U.L.D. lays heavily on the relation

\[ (\text{Mod}) \exists x . \phi x := (\exists x) \text{Mod} \phi x \]

As far as I know, Quine’s objections concerning equivalences of the form

“(\text{Mod})” = “a modal operator whatever”
“(\text{Quan})” = “a quantifier whatever”

\[ (\text{Quan}). (\text{Mod}) \phi x := (\text{Mod}). (\text{Quan}). \phi x \]

have not yet received an unanimously accepted solution.

Forbes’ operator “\( \Sigma x \)” allows to avoid the use of the modal operator of possibility because said symbol stands for “for at least one possible \( x \)” ...”, and his use of “\( \exists x \)” to denote “for at least one real \( x \)” allows to distinguish real from fictional existence without the help of the operator of necessity; this allows to evaluate sentences concerning objects of U.L.D. without out stepping from non modal to modal logic any time it is needed to state if said objects are fictional or real. As already shown in par. 3), there are

\[^{20} \text{Dicto for real objects.}\]
alternative ways to distinguish fictional from non fictional objects without making use of modal operators. Yet, these, and any other alternative way to distinguish fictional from non fictional existence without by-passing the boundaries of amodal (non modal) logic are, how to say, "rivers from the same source": Forbes’ concept of existence as being part of a possible world. What many appear not equally acceptable is Forbes’ identification of possible existence as a mutually exclusive alternative between fictional and real existence; this because U.L.D. contains some objects of thought, like mathematical or logical objects, concepts of science and so on, the existence of which appears problematically be assumed as real or as fictional. Let us consider, for instance, the sentence

«If “3” is a number, then something is a number»

and let us suppose that this sentence is emitted in a contest such as to request to specify what kind of existence may be ascribed to numbers. Should the answer be “real existence”, it could look as numbers where being ascribed a kind of existence not qualitatively different, from an ontological point of view, to the kind of existence commonly ascribed to physical objects. On the other hand, to say that numbers exist fictionally could look like these objects of thought were ascribed the same kind of existence commonly ascribed to fictional objects whose existence is originally due to imagination rather then rationality.

This problem could be solved by defining fictional existence in terms such as it could be ascribed to all mental (non physical) objects. In Forbes 1989 such a definition is missing; allow me to leave the reader “the honour and the onus” to solve this problem.

References

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