On the Unique Formulation of the Principles of Identity, Non-Contradiction and Excluded Middle

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Introduction - First Part

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INTRODUCTION

It is possible to demonstrate that, in spite of three different graphic shape, the Principles of Identity, Non-Contradiction and Excluded Middle have one and only one and the same logical sense. In other words, whereas in pure syntax it is a question of three different expressions, given that the three Principles are made up of unlike spoken or written signs, in pure semantics it is matter of one and only one and the same logic meaning. We use ‘meaning’ as sinonimous of ‘sense’.

We know that the three Principles have weak and stark formulations. Now we have to make progress. We must demonstrate that:

a) if we start from the definition of material implication, or conditional, and employ some rules of deduction, then we achieve a unique formulation of the three Principles in weak form;
b) if we start from the definition of converse material implication, or converse conditional, and employ the same rules of deduction, then we achieve a unique formulation of the three Principles in weak form;  
c) if we start from the definition of material equivalence, or biconditional, and from the definition of material non-

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equivalence, or exclusive disjunction, and employ the same rules of deduction plus a supplementary rule, then we achieve a unique formulation of the three Principles in stark form.

We know also that either material implication or converse material implication are definable by negation and logical sum or by negation and logical product. We shall follow both the ways.

Nevertheless, in order to avoid running the risk some mistake can creep into the course of the work, we shall construct a language $L = \ldots$ in form of calculus, which we shall interpret only at the end.

This work will be divided in three parts: syntax of the language $L = \ldots$, relative calculus, formulation of some problems (FIRST PART); transformation of Boole's algebra into Sheffer's algebra, which is a *sui generis* boolean algebra (SECOND PART); relations between the first two parts, semantics of the language $L = \ldots$ and solution of the problems put in the first part (THIRD PART). A set of philosophical remarks will conclude the work.

**FIRST PART**

**THE LANGUAGE $L =$**

The language $L =$ is a language set up by symbolic expressions, in each of them the signe ‘=’ shows up. The sign ‘=’ sets a relation among expressions of the language $L$, which are the elements of $L = \ldots$. Therefore we must formulate syntax of language $L$ before syntax of language $L =$

**1.1 Syntax of Language $L$**
Symbols of language $L$

Variables: $p, q$

Connectives:
  a) monadic: $N$
  b) dyadic: $A, B, C, D, E, J, K, X$

Rules of formation:
  - $p$ is a well formed formula;
  - $q$ is a well formed formula;
  - if $a$ is a well formed formula, then $Na$ is a well formed formula;
    - if $\alpha$ is a well formed formula and $\beta$ is a well formed formula, then $A\alpha\beta$ is a well formed formula;
    - if $\alpha$ is a well formed formula and $\beta$ is a well formed formula, then $B\alpha\beta$ is a well formed formula;
    - if $\alpha$ is a well formed formula and $\beta$ is a well formed formula, then $C\alpha\beta$ is a well formed formula;
    - if $\alpha$ is a well formed formula and $\beta$ is a well formed formula, then $D\alpha\beta$ is a well formed formula;
    - if $\alpha$ is a well formed formula and $\beta$ is a well formed formula, then $E\alpha\beta$ is a well formed formula;
    - if $\alpha$ is a well formed formula and $\beta$ is a well formed formula, then $J\alpha\beta$ is a well formed formula;
    - if $\alpha$ is a well formed formula and $\beta$ is a well formed formula, then $K\alpha\beta$ is a well formed formula;
    - if $\alpha$ is a well formed formula and $\beta$ is a well formed formula, then $X\alpha\beta$ is a well formed formula.

No expression is a well formed formula unless its being so follows from the above rules.

Decidability
Let $Z$ stand for any well formed formula. The following symbolic equalities are then satisfied:

\[
\begin{align*}
  p &= Z \\
  q &= Z \\
  NZ &= Z \\
  AZZ &= Z \\
  EZZ &= Z \\
  BZZ &= Z \\
  JZZ &= Z \\
  CZZ &= Z \\
  KZZ &= Z \\
  DZZ &= Z \\
  XZZ &= Z
\end{align*}
\]

E. g. let us control the formula $CKpqNApq$. We replace in this formula every variable by $Z$ and then apply the symbolic equalities given above:

\[
CKpqNApq = CKZZNAZZ = CZNZ = CZZ = Z.
\]

Since the result has been one and only one $Z$, then the above formula is a well formed formula. In general, given any formula at will, if, after replacing in it all the variables with $Z$ and applying the above symbolic equalities, the result is one and only one $Z$, then the expression is a well formed formula, otherwise it is not.

### 1.2 Syntax of the language $L =$

**Symbols:**

a) all the symbols of the language $L$ belong to the language $L =$;  
b) the sign ‘$=$’ which is read “is the same as” belongs to the language $L =$.

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**Rules of formation**

If $\alpha$ is a well formed formula of the language $L$ and $\beta$ is a well formed formula of the language $L$, then $\alpha = \beta$ is a correct expression of the language $L = \alpha$.

**Decidability**

An expression of language $L = \alpha$ is correct if and only if $Z = Z$.

E.g. let us verify the correctness of the expression

$Cpp = NANANApppNANAppp$.

We substitute every variable with $Z$, then we go on as follows:

$CZZ = NANANAZZZNANAZZZ ; Z = NANANZZNANZZ ; Z = NANAZZNAZZ ; Z = NANZNZ ; Z = NZ ; Z = Z.$

Therefore the expression $CZZ = NANANAZZZNANAZZZ$ is correct in language $L = \alpha$.

**Rules of deduction**

Let $\alpha, \beta, \gamma, \delta$ be signs of metalanguage of language $L = \alpha$ i.e. signs which refer to well formed formulas of language $L$.

Let English be metalanguage of $L = \alpha$.

**Rule I (R. I)**

If it is asserted that $\alpha$ is the same as $\beta$, then if $\text{in } \alpha$ and in
β a variable γ is substituted in every occurrence with a well formed formula δ, then it can be asserted that α in which γ has been substituted in every occurrence with δ is the same as β in which γ has been substituted in every occurrence with δ.

Rule II a (R.II a)

If it is asserted that α is the same as β, then if in β an expression γ, which can coincide with all β too, is substituted with NNγ, then it can be asserted that α is the same as β in which γ has been substituted with NNγ.

Rule II b (R.II b)

If it is asserted that α is the same as β, then if in β an expression NNγ, which can coincide with all β too, is substituted with γ, then it can be asserted that α is the same as β in which NNγ has been substituted with γ.

Rule III a (R.III a)

If it is asserted that α is the same as β, then if in β an expression NAγδ, which can coincide with all β too, is substituted with KNγNδ, then it can be asserted that α is the same as β in which NAγδ has been substituted with KNγNδ.

Rule III b (R.III b)

If it is asserted that α is the same as β, then if in β an expression NKγδ, which can coincide with all β too, is substituted with ANγNδ, then it can be asserted that α is the same as β in which NKγδ has been substituted with ANγNδ.

Rule IV a (R.IV a)

If it is asserted that α is the same as β, then if in β an expression γ, which can coincide with all β too, is substituted with Kγγ, then it can be asserted that α is the same as β in which γ has been substituted with Kγγ.

Rule IV b (R.IV b)

If it is asserted that α is the same as β, then if in β an expression γ, which can coincide with all β too, is substituted with Aγγ, then it can be asserted that α is the same as β in which γ has been substituted with Aγγ.

Rule V a (R.V a)
If it is asserted that $\alpha$ is the same as $\beta$, then if in $\beta$ an expression $NK\gamma$ has been substituted with $D\gamma$, then it can be asserted that $\alpha$ is the same as $\beta$ in which $NK\gamma$ has been substituted with $D\gamma$.

Rule V b (R.V b)

If it is asserted that $\alpha$ is the same as $\beta$, then if in $\beta$ an expression $NA\gamma$ has coincided with all $\beta$ too, is substituted with $X\gamma$, then it can be asserted that $\alpha$ is the same as $\beta$ in which $NA\gamma$ has been substituted with $X\gamma$.

Rule V c (R.V c)

If it is asserted that $\alpha$ is the same as $\beta$, then if in $\beta$ an expression $ND\gamma$ has coincided with all $\beta$ too, is substituted with $XN\gamma$ and then $\gamma$ can be asserted that $\alpha$ is the same as $\beta$ in which $ND\gamma$ has been substituted with $XN\gamma$.

Rule VI (R.VI)

If it is asserted that $\alpha$ is the same as $\beta$, and it is asserted that $\alpha$ is the same as $\gamma$, then it can be asserted that $\beta$ is the same as $\gamma$.

Rule VII (R.VII)

If it is asserted that $\alpha$ is the same as $\beta$, and it is asserted that $\gamma$ is the same as $\beta$, and it is asserted that $\gamma$ is the same as $\delta$, then it can be asserted that $\alpha$ is the same as $\delta$.

Rule VIII (R.VIII)

If it is asserted that $\alpha$ is the same as $\beta$, then if $\gamma$ is a part of $\beta$ and it is asserted that $\gamma$ is the same as $\delta$, then it can be asserted that $\alpha$ is the same as $\beta$ in which $\gamma$ has been substituted by $\delta$.

1.1.1 First group of theorems

Definitions

0.1 $Cpq = ANpq$ Df.
0.2 $Cpq = NKpNq$ Df.

Deduction

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Theorem I a

\[ Cpp = DpDpp \]

Proof

(1) \( Cpp = ANpp \) (by 0.1; \( q/p \), R.I)
(2) \( = NNANpp \) (by (1); R.II a)
(3) \( = NKKNpNp \) (by (2); R.III a)
(4) \( = NKpNp \) (by (3); R.II b)
(5) \( = DpNp \) (by (4); R.V a)
(6) \( = DpNKpp \) (by (5); R.IV a)
(7) \( = DpDpp \) (by (6); R.V a)

Theorem II a

\[ NKpNp = DpDpp \]

Proof

(1) \( NKpNp = DpNp \) (by (4) and (5) Th.I a; R.VI)
(2) \( = DpNKpp \) (by (1) Th.II a, (5) and
       (6) Th.I a; R.VII)
(3) \( = DpDpp \) (by (2) Th.II a, (6) and
       (7) Th.I a; R.VII)

Theorem III a

\[ ANpp = DpDpp \]

Proof

(1) \( ANpp = NNANpp \) (by (1) and (2) Th.I a; R.VI)
(2) \( = NKKNpNp \) (by (1) Th.III a, (2) and
       (3) Th.I a; R.VII)
(3) \( = NKpNp \) (by (2) Th.III a, (3) and
       (4) Th.I a; R.VII)
(4) \( = DpNp \) (by (3) Th.III a, (4) and
Theorem IV a

\[ \text{DpNp = DpDpp} \]

Proof

(1) \[ \text{DpNp = DpNKpp} \] (by (5) Th.I a; R.VII)

(2) \[ \text{DpDpp} \] (by (1) Th.IV a; (6) Th.I a; R.VII)

Theorem I b

\[ \text{Cpp = XXXpppXXppp} \]

Proof

(1) \[ \text{Cpp = NKpNp} \] (by 0.2; q/p, R.I)

(2) \[ \text{ANpNNp} \] (by (1) R.III b)

(3) \[ \text{ANpp} \] (by (2); R.II b)

(4) \[ \text{NANANpp} \] (by (3); R.II a)

(5) \[ \text{NANANAppp} \] (by (4); R.IV b)

(6) \[ \text{NANANApppNANAppp} \] (by (5); R.IV b)

(7) \[ \text{XXXpppXXppp} \] (by (6); R.V b, 5 times)

Theorem II b

\[ \text{NKpNp = XXXpppXXppp} \]

Proof

(1) \[ \text{NKpNp = ANpNNp} \] (by (1) and (2) Th.I b; R.VI)

(2) \[ \text{ANpp} \] (by (1) Th.II b, (2) and...
(3) Th. I b; R. VII

(3) \( = \text{NNANpp} \) (by (2) Th. II b, (3) and
(4) Th. I b; R. VII)

(4) \( = \text{NNANAppp} \) (by (3) Th. II b, (4) and
(5) Th. I b; R. VII)

(5) \( = \text{NANANAppppNANApppp} \) (by (4) Th. III b, (5)
and (6) Th. I b; R. VII)

(6) \( = \text{XXXpppXXppp} \) (by (5) Th. II b, (6) and
(7) Th. I b; R. VII)

*Theorem III b*

\( \text{ANpp} = \text{XXXpppXXppp} \)

*Proof*

(1) \( \text{ANpp} = \text{NNANpp} \) (by (3) and (4) Th. I b;
R. VI)

(2) \( = \text{NNANAppp} \) (by (1) Th. III b, (4) and
(5) Th. I b; R. VII)

(3) \( = \text{NANANAppppNANApppp} \) (by (2) Th. III b, (5)
and (6) Th. I b; R. VII)

(4) \( = \text{XXXpppXXppp} \) (by (3) Th. III b, (6) and
(7) Th. I b; R. VII)

*Lemma I b 1*

\( \text{Cpp} = \text{NNDpNp} \)

*Proof*

(1) \( \text{Cpp} = \text{NKpNp} \) (by 0.2; R. I)

(2) \( = \text{DpNp} \) (by (1); R. V a)

(3) \( = \text{NNDpNp} \) (by (2); R. II a)

*Theorem IV b*

\( \text{DpNp} = \text{XXXpppXXppp} \)
Proof
(1) \( DpNp = NNDpNp \) (by (2) and (3) Lem.I b 1; R.VI)
(2) \( = NXNpNNp \) (by (1) ; R.V c)
(3) \( = NXNpp \) (by (2) ; R.II b)
(4) \( = NXNAppp \) (by (3) ; R.IV b)
(5) \( = NXXppp \) (by (4) ; R.V b)
(6) \( = NAXXpppXXppp \) (by (5) ; R.IV b)
(7) \( = XXXpppXXppp \) (by (6) ; R.V b)

1.2.2 Second group of theorems

Definitions

0.3 \( Bpq = ApNq \)
0.4 \( Bpq = NKNpq \)

Deduction

Theorem I c

\( Bpp = DDppp \)

Proof
(1) \( Bpp = ApNp \) (by 0.3; q /p, R.I)
(2) \( = NNApNp \) (by (1); R.II a)
(3) \( = NKNpNNp \) (by (2); R.III a)
(4) \( = NKNpp \) (by (3); R.II b)
(5) \( = DNpp \) (by (4); R.V a)
(6) \( = DNKppp \) (by (5); R.IV a)
(7) \( = DDppp \) (by (6); R.V a)

Theorem II c

\( NKNpp = DDppp \)

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Proof
(1) \( NKNpp = DNpp \)  
(by (4) and (5) Th.I c; R.VI)
(2) \( = DNKppp \)  
(by (1) Th.II c, (5) and  
(6) Th.I c; R.VII)
(3) \( = DDppp \)  
(by (2) Th.II c, (6) and  
(7) Th.I c; R.VII)

Theorem III c

Proof

\( ApNp = DDppp \)

(1) \( ApNp \)  
(by (1) and (2) Th.I c; R.VI)
(2) \( = NNApNp \)  
(by (1) Th.III c, (2) and (3) Th.I c;  
R.VII)
(3) \( = NKNpp \)  
(by (2) Th.III c, (3) and (4) Th.I c;  
R.VII)
(4) \( = DNpp \)  
(by (3) Th.III c, (4) and (5) Th.I c;  
R.VII)
(5) \( = DNKppp \)  
(by (4) Th.III c, (5) and (6) Th.I c;  
R.VII)
(6) \( = DDppp \)  
(by (5) Th.III c, (6) and (7) Th.I c;  
R.VII)

Theorem IV c

\( DNpp = DDppp \)

Proof

(1) \( DNpp \)  
(by (5) and (6) Th.I c; R.VI)
(2) \( = DNKppp \)  
(by (1) Th.IV c, (6) and (7) Th.I c;  
R.VII)

Theorem I d

\( Bpp = XXpXppXpXpp \)

Proof
Theorem II d

\[ NKNpp = XXpXppXpXpp \]

Proof

(1) \[ NKNpp = ANNpNp \] (by (1) and (2) Th.I d; R.VI)
(2) \[ = ApNp \] (by (1) Th.II d, (2) and (3) Th.I d; R.VII)
(3) \[ = NNApNp \] (by (2) Th.II d, (3) and (4) Th.I d; R.VII)
(4) \[ = NNApNApApp \] (by (3) Th.II d, (4) and (5) Th.I d; R.VII)
(5) \[ = NANApNApAppNApp \] (by (4) Th.II d, (5) and (6) Th.I d; R.VII)
(6) \[ = XXpXppXpXpp \] (by (5) Th.II d, (6) and (7) Th.I d; R.VII)

Theorem III d

\[ ApNp = XXpXppXpXpp \]

Proof

(1) \[ ApNp = NNApNp \] (by (3) and (4) Th.I d; R.VI)
(2) \[ = NNApNApp \] (by (1) Th.III d, (4) and (5) Th.I d; R.VII)
(3) \[ = NANApNAppNApp \] (by (2) Th.III d, (5) and (6) Th.I d; R.VII)
(4) \[ = XXpXppXpXpp \] (by (3) Th.III d, (6) and (7) Th.I d; R.VII)

Lemma I d 1

\[ Bpp = NNDNpp \]

Proof

1. \[ Bpp = NKNpp \] (by 0.4; q /p, R.I)
2. \[ = DNpp \] (by (1); R.V a)
3. \[ = NNDNpp \] (by (2); R.II a)

Theorem IV d

\[ DNpp = XXpXppXpXpp \]

Proof

1. \[ DNpp = NNDNpp \] (by (2) and (3) Lem.I d 1; R.VI)
2. \[ = NXNNpNp \] (by (1); R.V c)
3. \[ = NXpNp \] (by (2); R.II b)
4. \[ = NXpNApp \] (by (3); R.IV b)
5. \[ = NXpXpp \] (by (4); R.V b)
6. \[ = NAXpXppXpXpp \] (by (5); R.IV b)
7. \[ = XXpXppXpXpp \] (by (6); R.V b)

1.2.3 Third group of theorems

Definitions

0.5 \[ Epq = KCpqBpq \]
0.6 \[ Jpq = KApqDpq \]

Deduction

Theorem I e

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\[ Epp = DDDpDppDDpppDDpDppDDppp \]

Proof

(1) \[ Epp = KCppBpp \] (by 0.5; \( q/p, \) R.I)
(2) \[ = KNKnNpBpp \] (by (1) Th.I e, (4) Th.I a; \( Cpp //NKnNp, \) R.VIII)
(3) \[ = KNKnNpNKnpp \] (by (2) Th.I e, (4) Th.I c; \( Bpp //NKnpp, \) R.VIII)
(4) \[ = KNKnKppNKnKppp \] (by (3) Th.I e; R.IV a, twice)
(5) \[ = KDpDppDDppp \] (by (4) Th.I e; R.V a, 4 times)
(6) \[ = NKnDpDppDDppp \] (by (5) Th.I e; R.II a)
(7) \[ = NDpDppDDppp \] (by (6) Th.I e; R.V a)
(8) \[ = NKDpDppDDpppDDpDppDDppp \] (by (7) Th.I e; R.IV a)
(9) \[ = DDDpDppDDpppDDpDppDDppp \] (by (8) Th.I e; R.V a)

Theorem II e

\[ KNKnNpNKnpp = DDDpDppDDpppDDpDppDDppp \]

Proof

(1) \[ KNKnNpNKnpp = KNKnKppNKnKppp \] (by (3) and (4) Th.I e; R.VI)
(2) \[ = KDpDppDDppp \] (by (1) Th.II e, (4) and (5) Th.I e; R.VII)
(3) \[ = NKnDpDppDDppp \] (by (2) Th.II e, (5) and (6) Th.I e; R.VII)
(4) \[ = NDpDppDDppp \] (by (3) Th.II e, (6) and (7) Th.I e; R.VII)
(5) \[ = NKDpDppDDpppDDpDppDDppp \] (by (4) Th.II e, (7) and (8) Th.I e; R.VII)
(6) \[ = DDDpDppDDpppDDpDppDDppp \] (by (5) Th.II e, (8) and
Theorem III e

\[ JNpp = DDDpDppDDpppDDpDppDDppp \]

Proof

(1) \[ JNpp = KANppDNpp \] (by 0.6; \( p \perp Np, q / p, \) R.I)
(2) \[ = KNNANppDNpp \] (by (1); R.II a)
(3) \[ = KNKNNpNpDNpp \] (by (2); R.III a)
(4) \[ = KNKpNpDNpp \] (by (3); R.II b)
(5) \[ = NNKNKpNpDNpp \] (by (4); R.II a)
(6) \[ = NKNKNNKpNpDNppNKNKpNpDNpp \] (by (5); R.IV a)
(7) \[ = NKNKKNKpNpDNppNKNKpNpDNpp \] (by (6); R.IV a, 4 times)
(8) \[ = DDDpDppDDpppDDpDppDDppp \] (by (7); R.Va, 9 times)

Theorem I f

\[ Epp = XXXpppXpXpp \]

Proof

(1) \[ Epp = KCppBpp \] (by 0.5; \( q / p, \) R.I)
(2) \[ = KNKpNpBpp \] (by (1) Th.I f, (1) Th.I b; \( Cpp \parallel NKpNp, \) R.VIII)
(3) \[ = KNKpNpNKNpp \] (by (2) Th.I f, (1) Th.I d; \( Bpp \parallel KNpNp, \) R.VIII)
(4) \[ = KANpNNpANNpNp \] (by (3) Th.I f; R.III b, twice)
(5) \[ = KANppApNp \] (by (4) Th.I f; R.II b)
(6) \[ = NNKANppApNp \] (by (5) Th.I f; R.II a)
(7) \[ = NANANpApNp \] (by (6) Th.I f; R.III b)
(8) \[ = NANANApppApNApp \] (by (7) Th.I f; R.IV b, twice)
(9) \[ = XXXpppXpXpp \] (by (8) Th.I f; R.V b, 5 times)
Theorem II f

\[ \text{KNKpNpNKNpp} = XXXpppXpXpp \]

**Proof**

1. \[ \text{KNKpNpNKNpp} = KANpNNpANpNpNp } \text{ (by (3) and (4)} \text{ Th.I f; R.VI) } \]
2. \[ = KANppApNp \text{ (by (1) Th.II f, (4) and (5) Th.I f; R.VII) } \]
3. \[ = NNNKANppApNp \text{ (by (2) Th.II f, (5) and (6) Th.I f; R.VII) } \]
4. \[ = NANANppNAPNpNp \text{ (by (3) Th.II f, (6) and (7) Th.I f; R.VII) } \]
5. \[ = NANANApppNAPNpNp \text{ (by (4) Th.III f (7) and (8) Th.I f; R.VII) } \]
6. \[ = XXXpppXpXpp \text{ (by (5) Th.II f, (8) and (9) Th.I f; R.VII) } \]

Theorem III f

\[ \text{JNpp} = XXXpppXpXpp \]

**Proof**

1. \[ \text{JNpp} = KANppDNpp \text{ (by 0.6; } p \text{ } /Np, q /p, \text{ R.I) } \]
2. \[ = NNNKANppDNpp \text{ (by (1); R.II a) } \]
3. \[ = NANANppNNDNpp \text{ (by (2); R.III b) } \]
4. \[ = NANANppXNNpNp \text{ (by (3); R.V c) } \]
5. \[ = NANANppXpNp \text{ (by (4); R.II b) } \]
6. \[ = NANANApppXpNApp \text{ (by (5); R.IV b, twice) } \]
7. \[ = XXXpppXpXpp \text{ (by (6); R.V b, 4 times) } \]

Conclusion

Starting by definition

0.1 \[ \text{Cpq} = ANpq \]
it has been demonstrated

\[
\begin{align*}
C_{pp} &= DpDpp \\
NK_{pNp} &= DpDpp \\
AN_{pp} &= DpDpp \\
Dp_{Np} &= DpDpp
\end{align*}
\]

and starting by definition

0.2 \quad C_{pq} = NK_{pNq}

it has been demonstrated

\[
\begin{align*}
C_{pp} &= XXXpppXXppp \\
NK_{pNp} &= XXXpppXXppp \\
AN_{pp} &= XXXpppXXppp \\
Dp_{Np} &= XXXpppXXppp
\end{align*}
\]

Hence

\[
DpDpp = XXXpppXXppp \quad \text{(It follows by Theorems I a - IV b).}
\]

Analogously starting by definition

0.3 \quad B_{pq} = Ap_{Nq}

it has been demonstrated

\[
\begin{align*}
B_{pp} &= DDppp \\
NK_{Npp} &= DDppp \\
Ap_{Np} &= DDppp \\
Dp_{Np} &= DDppp
\end{align*}
\]

and starting by definition

0.4 \quad B_{pq} = NK_{Npp}
it has been demonstrated

\[
\begin{align*}
Bpp & = XXpXppXpXpp \\
NKNpp & = XXpXppXpXpp \\
ApNp & = XXpXppXpXpp \\
DNpp & = XXpXppXpXpp.
\end{align*}
\]

Hence

\[
DDppp = XXpXppXpXpp \quad (\text{It follows by theorems I c - IV d}).
\]

Lastly, starting by definitions

\[
\begin{align*}
0.5 & \quad Epq = KCpqBpq \\
0.6 & \quad Jpq = KApqDpq
\end{align*}
\]

it has been demonstrated

\[
\begin{align*}
Epp & = DDDpDppDDpppDDpDppDDppp \\
KNKpNpNKNpp & = DDDpDppDDpppDDpDppDDppp \\
JNpp & = DDDpDppDDpppDDpDppDDppp
\end{align*}
\]

aand it has been demonstrated

\[
\begin{align*}
Epp & = XXXpppXpXpp \\
KNKpNpNKNpp & = XXXpppXpXpp \\
JNpp & = XXXpppXpXpp
\end{align*}
\]

Hence

\[
DDDpDpp-DDppp-DDppp-DDpDpp-DDppp = XXXpppXpXpp \quad (\text{It follows by theorems I e - III f}).
\]

Let us call \( D \)-formulas the expressions of \( L \) in which the only connective \( D \) appears and \( X \)-formulas the expressions of \( L \) in which the only connective \( X \) appears.

Some problems require to be solved:
i) we must justify the above demonstrating process and we must interpret the expressions of language $L$;

ii) we must examine the relations between $D$-formulas obtained in theorems I a - IV a and $D$-formulas obtained in theorems I c - IV c, then their relations with $D$-formulas obtained in theorems I e - III e; in the same way we must examine the relations between $X$ formulas obtained in theorems I b - IV b and $X$-formulas obtained in theorems I d - IV d, then their relations with $X$-formulas obtained in theorems I f - III f;

iii) we must see if there are $D$-formulas which are the same as $DDDpDppDDpDppDDpDppDDpDppDDpDppDDpDppDDpDppDDpDppDDpDppDDpDpp$ and if there are $X$-formulas which are the same as $XXXpppXpXpp$;

iv) finally we must analyse the relations between $D$-formulas and $X$-formulas.